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A NOTE ON THE POINTING ERROR IMPACT ON THE RECOVERY OF THE STOKES PARAMETERS OF CMB POLARIZATION ANISOTROPY

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SUMMARY - Although the Low Frequency Instrument onboard PLANCK satellite has been design to measure the anisotropies of the Cosmic Microwave Background, its radiometers are intrinsecally able to measure the radiation polarization parameters with a particularly good sensitivity in the regions close to the ecliptic poles. A good knowledge of the polarization properties of each main beam and a high accuracy on the knowledge of the spacecraft pointing direction are required to reach this aim. PLANCK/LFI polarization measurements will be obtained by combining the signal received by two feed horns appropriately aligned in the focal plane. As a conseguence the accuracy of the telescope pointing direction in rotation is crucial because of its relationship with the main beam orientation in the sky. In this report we estimate the impact of the uncertainty in pointing direction around the telescope line of sight on the measurement of the Stokes parameters Q and U.

1 Introduction

The Low Frequency Instrument (LFI) onboard PLANCK satellite has been design to measure the anisotropies of the Cosmic Microwave Background (CMB). Together with the High Frequency Instrument (HFI), LFI will observe the full sky twice from a Lissajous orbit around the Lagrangian point L2 and will produce unprecedented angular resolution and sensitivity maps (see [1], [2]). Although LFI has been design to measure the CMB temperature anisotropies, its radiometers are intrinsecally able to measure the radiation polarization parameters.

The baseline layout of LFI Focal Plane Unit (FPU) foresees 23 feed horns located around HFI: 2 feed horns at 30 GHz, 3 at 44 GHz, 6 at 70 GHz and 12 at 100 GHz. All LFI feed horns are off-axis, and the respective main beams are located at 3 to 5 degrees from the telescope pointing direction, named Line Of Sight (LOS). LFI polarization properties have been optimized for polarization measurements rotating each front end module around the feed horn axis, in order to obtain the most appropriate orientation of each main beam [3]. The accuracy in the knowledge of the satellite position regard to a rotation around the telescope pointing direction is crucial for polarization measurements being necessary for a right knowledge of the polarization direction of each main beam and then for a correct estimation of the Stokes parameters Q and U. An accuracy on the reconstructed knowledge of the telescope pointing direction in rotation (AME in rotation[1]) better than $5'$ at 1σ is required to avoid significant degradations in the CMB temperature anisotropy measurement [4]. In this report we present a first estimation of the impact of the pointing direction uncertainty on the determination of the Stokes parameters Q and U.

2 Polarization Measurements with PLANCK/LFI

The LFI polarization measurements of the CMB will be performed by using its radiometers as differencing polarimeters [5]. LFI is an array of 46 HEMT-based pseudo-correlation receivers coupled to the Telescope by 23 dual profiled corrugated feed horns. The radiation gathered by each feed horn is divided into two orthogonal linearly polarized signals by an Ortho Mode Transducer. Each signal is joined to a radiometer where it is mixed by an hybrid coupler with a reference signal coming from a 4K load thermally connected to the HFI shield. The two output branches of the hybrid coupler are amplified by a couple of cryogenically cooled HEMTs low noise amplifiers, appropriately shifted in phase and in a synchronous way with the data acquisition, decoupled by another hybrid in order to remove common noise, and sent to a back end section where they are filtered, detected, and differenced [6].

Therefore, with one feed horn two Stokes parameters can be reconstructed: I and Q, or I and U, or I and a combination of Q and U, according to equations (1) - (6). Since the elliptical polarization of LFI main beams is less than 1%, they can be considered linearly polarized. Using two feed horns appropriately aligned in the focal plane, so that the main beam polarization directions form an angle equal to 45 degrees (when they observe the same direction in the sky), I, Q, and U can be measured [7].

Let I_{11} and I_{12} the responses of the two radiometers connected to the feed horn 1 and I_{21} and I_{22} the responses of the two radiometers connected to the feed horn 2. Then:

$$I_{11} = \frac{1}{2}\cdot[I + Q\cdot\cos(2\phi_{11}) + U\cdot\sin(2\phi_{11})] \quad (1)$$

$$I_{12} = \frac{1}{2}\cdot[I - Q\cdot\cos(2\phi_{11}) - U\cdot\sin(2\phi_{11})] \quad (2)$$

$$I_{21} = \frac{1}{2}\cdot[I + Q\cdot\cos(2\phi_{21}) + U\cdot\sin(2\phi_{21})] \quad (3)$$

$$I_{22} = \frac{1}{2}\cdot[I - Q\cdot\cos(2\phi_{21}) - U\cdot\sin(2\phi_{21})]\,. \quad (4)$$

Assuming that $\phi_{21} = \phi_{11} + 45°$ and redefining $\phi_{11} = \phi$, we derive the measured Stokes parameters, Q^m and U^m, by inverting equations (1) - (4) :

$$Q^m = (I_{11} - I_{12})\cdot\cos(2\phi) - (I_{21} - I_{22})\cdot\sin(2\phi) \quad (5)$$

$$U^m = (I_{21} - I_{22})\cdot\cos(2\phi) + (I_{11} - I_{12})\cdot\sin(2\phi)\,; \quad (6)$$

[1]The Attitute Measurement Error (AME) is defined as the instantaneous angular separation between the actual and the measured pointing direction. This performance requirement is referred to as *a posteriori* knowledge.

here ϕ is properly the angle between the major axis of polarization ellipse of the main beam coupled to the first feed horn and a defined direction in the sky, namely that of the Galactic meridian at the considered pointing direction. Neglecting receiver noise and any kind of systematic effects, in absence of pointing errors, we have that $Q^m = Q$ and $U^m = U$, where Q and U are the true values of the Stokes parameters in the sky (according to the chosen reference system, i.e. the Galactic reference in the present note).

3 Effect of a pointing error on the Stokes parameters

A satellite rotation around the telescope boresight direction involves an analogous rotation of the main beam polarization direction in the sky, as reported in Appendix. If an uncertainty in the pointing direction around LOS occurs ($\delta\phi_{11}$ for the feed horn 1 and $\delta\phi_{21}$ for the feed horn 2), the responses of the four considered radiometers are given by:

$$I_{11} = \frac{1}{2} \cdot \{I + Q \cdot \cos[2(\phi_{11} + \delta\phi_{11})] + U \cdot \sin[2(\phi_{11} + \delta\phi_{11})]\} \quad (7)$$

$$I_{12} = \frac{1}{2} \cdot \{I - Q \cdot \cos[2(\phi_{11} + \delta\phi_{11})] - U \cdot \sin[2(\phi_{11} + \delta\phi_{11})]\} \quad (8)$$

$$I_{21} = \frac{1}{2} \cdot \{I + Q \cdot \cos[2(\phi_{21} + \delta\phi_{21})] + U \cdot \sin[2(\phi_{21} + \delta\phi_{21})]\} \quad (9)$$

$$I_{22} = \frac{1}{2} \cdot \{I - Q \cdot \cos[2(\phi_{21} + \delta\phi_{21})] - U \cdot \sin[2(\phi_{21} + \delta\phi_{21})]\}, \quad (10)$$

and the measured values Q^m and U^m are related to the true values, Q and U, by relations:

$$Q^m = F_{QQ} \cdot Q + F_{QU} \cdot U \quad (11)$$

$$U^m = F_{UQ} \cdot Q + F_{UU} \cdot U. \quad (12)$$

In particular, in the above equations, F_{QU} (F_{UQ}) is a multiplicative factor representative of contamination due by U (Q) on Q^m (U^m) while F_{QQ} (F_{UU}) gives the fraction of the true Stokes parameter Q (U) that properly contributes to the measured Stokes parameter Q^m (U^m). The factors F_{QU}, F_{UQ}, F_{QQ} and F_{UU} can be explicitely written as:

$$F_{QQ} = \cos(2\phi) \cdot \sin(2\phi) \cdot [\sin(2\delta\phi_{21}) - \sin(2\delta\phi_{11})] + \cos(2\delta\phi_{11}) + \sin^2(2\phi) \cdot [\cos(2\delta\phi_{21}) - \cos(2\delta\phi_{11})]$$

$$F_{QU} = \cos(2\phi) \cdot \sin(2\phi) \cdot [\cos(2\delta\phi_{11}) - \cos(2\delta\phi_{21})] + \sin(2\delta\phi_{11}) + \sin^2(2\phi) \cdot [\sin(2\delta\phi_{21}) - \sin(2\delta\phi_{11})]$$

$$F_{UQ} = \cos(2\phi) \cdot \sin(2\phi) \cdot [\cos(2\delta\phi_{11}) - \cos(2\delta\phi_{21})] - \sin(2\delta\phi_{21}) + \sin^2(2\phi) \cdot [\sin(2\delta\phi_{21}) - \sin(2\delta\phi_{11})]$$

$$F_{UU} = \cos(2\phi) \cdot \sin(2\phi) \cdot [\sin(2\delta\phi_{11}) - \sin(2\delta\phi_{21})] + \cos(2\delta\phi_{21}) + \sin^2(2\phi) \cdot [\cos(2\delta\phi_{11}) - \cos(2\delta\phi_{21})]$$

Fig. 1 shows the contamination levels of the Stokes parameter Q owing to U, considering several levels of uncertainty in the pointing direction around LOS. For the small pointing uncertainties relevant in this context, this contamination increases approximately linearly with the pointing error, as can be simply understood by expanding in powers to the first order in $\delta\phi_{11} = \delta\phi_{21}$ ($= \delta\phi$) the above factors (for example $F_{QU} = \sin(2\delta\phi) \to 2\delta\phi$ for $\delta\phi \to 0$). Assuming an uncertainty in the pointing direction around LOS of about $\pm 6'$ ($\delta\phi_{11} = 6'$ and $\delta\phi_{21} = -6'$), a contamination peaking of about 0.35% U is present in the measurement of Q. An analogous consideration holds for the measurement of U.

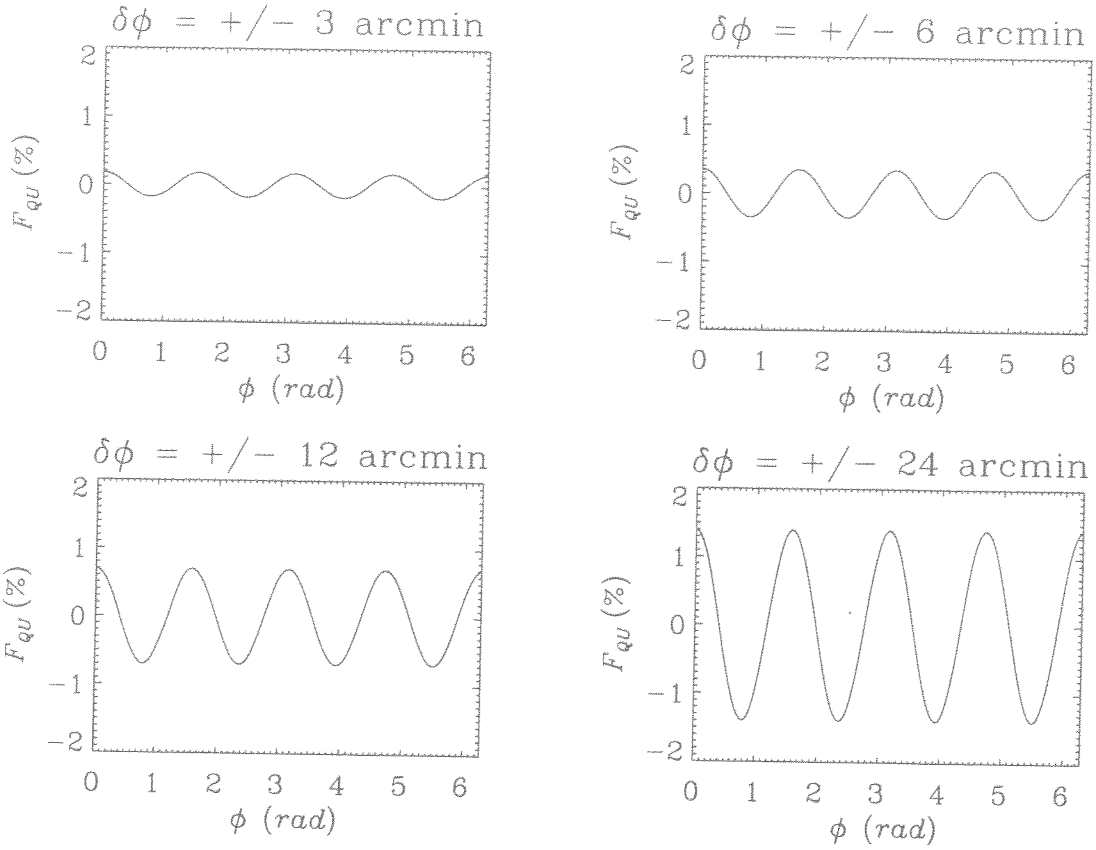

Figure 1: The factor F_{QU} representative of the contamination due by the Stokes parameter U on Q^m. Curves for F_{UQ}, $F_{QQ} - 1$, and $F_{UU} - 1$ are similar except for appropriate phase shifts.

Considering maps of Q and U for a typical cosmological model in agreement with the current CMB anisotropy observations and with Gaussian fluctuations, we can evaluate the percentage of pixels in the sky in which a certain value of the $|U/Q|$ ratio (see Fig. 2 for its distributions) implies a given error in the determination of Q owing to U. For example, a relative error of $\simeq 10\%$ ($\simeq 5\%$) is introduced in $\simeq 1\%$ ($\simeq 2\%$) of the sky pixels, as can be simply understood by observing that, for $F_{QQ} = 0$, $F_{QU} = 0.0035$, and U/Q $\simeq 30$ (15), equation (11) gives $Q^m/Q = F_{QQ} + F_{QU} \cdot U/Q \simeq 0.1$ (0.05).

To be more quantitative, we have computed an estimate of the upper limit overall relative error in the Q measurement considering a value of ϕ in which the function $|F_{QQ} - 1| + |F_{QU}|$ has a maximum for the representative case $\delta\phi_{11} = -\delta\phi_{21} = 6'$. For this value of ϕ (=10.7945°), we have computed the differential and cumulative distribution function of the quantity $\log[(|F_{QQ} - 1| \cdot |Q| + |F_{QU}| \cdot |U|)/|Q|]$, reported in Fig. 3. The percentage of pixels in which the overall relative error is greater than $\simeq 10\%$ ($\simeq 5\%$) is about 1.7% (3.4%), according to the previous estimate, and can still be considered negligible.

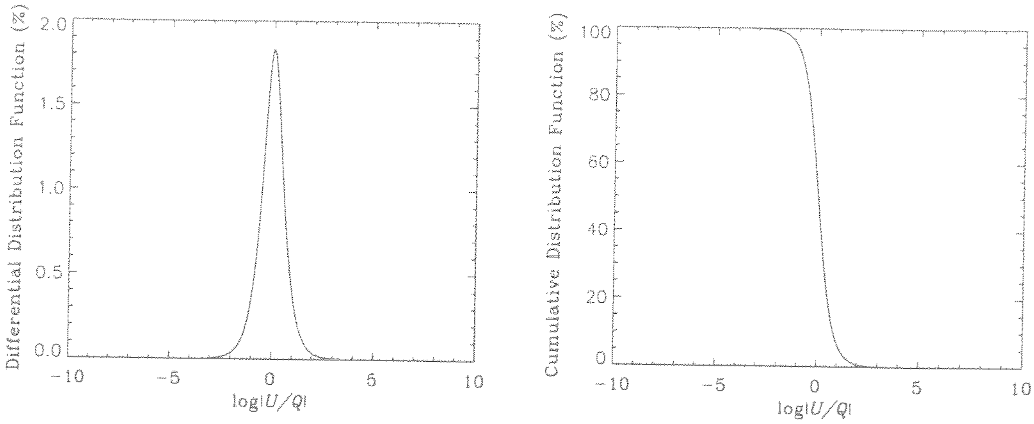

Figure 2: Differential and cumulative distribution function of $\log|U/Q|$.

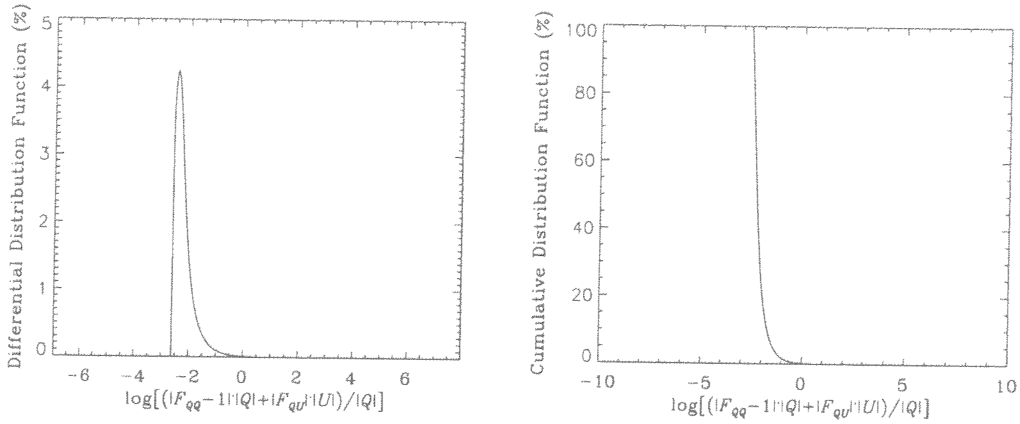

Figure 3: Differential and cumulative distribution function of the quantity $\log\left[(|F_{QQ}-1|\cdot|Q|+|F_{QU}|\cdot|U|)/|Q|\right]$. A similar result has been obtained for the overall relative error in the U estimation.

4 Conclusions

From the study previously reported, we can assert that an accuracy on the reconstructed knowledge of the telescope pointing direction in rotation better than $5'$ at 1σ (AME requirement for PLANCK/LFI) is sufficient to guarantee an adeguate reconstruction of the cosmic microwave background polarization properties.

Acknowledgements

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6 Appendix

Each PLANCK/LFI main beam is computed in its own coordinate system defined starting from the telescope Line Of Sight (LOS) frame [8], whose orientation (represented by θ_{MB} and ϕ_{MB} angles) gives the peak of directivity in $(U,V)_{MB} = (0,0)$ and X–axis is aligned with the main beam polarization direction (represented by the ψ_{MB} angle). According to the GRASP8 definition of the three angles θ, φ, and ψ [9], the components of the $\overline{X}_{MB}$, $\overline{Y}_{MB}$, $\overline{Z}_{MB}$ unit vectors in the $\overline{X}_{LOS}$, $\overline{Y}_{LOS}$, $\overline{Z}_{LOS}$ base are:

$$
\begin{aligned}
&\overline{X}_{MB}\cdot\overline{X}_{LOS} = \sin\varphi(\cos\psi\sin\varphi-\cos\varphi\sin\psi)-\cos\theta\cos\varphi(-\cos\varphi\cos\psi-\sin\varphi\sin\psi)\\
&\overline{X}_{MB}\cdot\overline{Y}_{LOS} = \cos\varphi(\cos\psi\sin\varphi-\cos\varphi\sin\psi)+\cos\theta\sin\varphi(-\cos\varphi\cos\psi-\sin\varphi\sin\psi)\\
&\overline{X}_{MB}\cdot\overline{Z}_{LOS} = -\sin\theta(-\cos\varphi\cos\psi-\sin\varphi\sin\psi)\\
&\overline{Y}_{MB}\cdot\overline{X}_{LOS} = -\cos\theta\cos\varphi(\cos\psi\sin\varphi-\cos\varphi\sin\psi)+\sin\varphi(\cos\varphi\cos\psi+\sin\varphi\sin\psi)\\
&\overline{Y}_{MB}\cdot\overline{Y}_{LOS} = \cos\theta\sin\varphi(\cos\psi\sin\varphi-\cos\varphi\sin\psi)+\cos\varphi(\cos\varphi\cos\psi+\sin\varphi\sin\psi)\\
&\overline{Y}_{MB}\cdot\overline{Z}_{LOS} = -\sin\theta(\cos\psi\sin\varphi-\cos\varphi\sin\psi)\\
&\overline{Z}_{MB}\cdot\overline{X}_{LOS} = -\cos\varphi\sin\theta\\
&\overline{Z}_{MB}\cdot\overline{Y}_{LOS} = \sin\theta\sin\varphi\\
&\overline{Z}_{MB}\cdot\overline{X}_{LOS} = \cos\theta.
\end{aligned}
$$

If an uncertainty, ξ, in the pointing direction around Z_{LOS} occurs, the components of the new rotated $\overline{X}_{MB\_rot}$, $\overline{Y}_{MB\_rot}$, $\overline{Z}_{MB\_rot}$ unit vectors in the LOS base are:

$$
\begin{aligned}
\overline{X}_{MB\_rot}\cdot\overline{X}_{LOS} &= \cos\xi[\sin\varphi(\cos\psi\sin\varphi-\cos\varphi\sin\psi)-\cos\theta\cos\varphi(-\cos\varphi\cos\psi-\sin\varphi\sin\psi)]+\\
&\quad+\sin\xi[\cos\varphi(\cos\psi\sin\phi-\cos\varphi\sin\psi)+\cos\theta\sin\varphi(-\cos\varphi\cos\psi-\sin\varphi\sin\psi)]\\
\overline{X}_{MB\_rot}\cdot\overline{Y}_{LOS} &= -\sin\xi[\sin\varphi(\cos\psi\sin\varphi-\cos\varphi\sin\psi)-\cos\theta\cos\varphi(-\cos\varphi\cos\psi-\sin\varphi\sin\psi)]+\\
&\quad+\cos\xi[\cos\varphi(\cos\psi\sin\varphi-\cos\varphi\sin\psi)+\cos\theta\sin\varphi(-\cos\varphi\cos\psi-\sin\varphi\sin\psi)]\\
\overline{X}_{MB\_rot}\cdot\overline{Z}_{LOS} &= -\sin\theta(-\cos\varphi\cos\psi-\sin\varphi\sin\psi)\\
\overline{Y}_{MB\_rot}\cdot\overline{X}_{LOS} &= \sin\xi[\cos\theta\sin\varphi(\cos\psi\sin\varphi-\cos\varphi\sin\psi)+\cos\varphi(\cos\varphi\cos\psi+\sin\varphi\sin\psi)]+\\
&\quad+\cos\xi[-\cos\theta\cos\varphi(\cos\psi\sin\phi-\cos\varphi\sin\psi)+\sin\varphi(\cos\varphi\cos\psi+\sin\varphi\sin\psi)]\\
\overline{Y}_{MB\_rot}\cdot\overline{Y}_{LOS} &= \cos\xi[\cos\theta\sin\varphi(\cos\psi\sin\varphi-\cos\varphi\sin\psi)+\cos\varphi(\cos\varphi\cos\psi+\sin\varphi\sin\psi)]+\\
&\quad-\sin\xi[-\cos\theta\cos\varphi(\cos\psi\sin\varphi-\cos\varphi\sin\psi)+\sin\varphi(\cos\varphi\cos\psi+\sin\varphi\sin\psi)]\\
\overline{Y}_{MB\_rot}\cdot\overline{Z}_{LOS} &= -\sin\theta(\cos\psi\sin\varphi-\cos\varphi\sin\psi)\\
\overline{Z}_{MB\_rot}\cdot\overline{X}_{LOS} &= -\cos\xi\cos\varphi\sin\theta+\sin\theta\sin\xi\sin\varphi\\
\overline{Z}_{MB\_rot}\cdot\overline{Y}_{LOS} &= \cos\varphi\sin\theta\sin\xi+\cos\xi\sin\theta\sin\varphi\\
\overline{Z}_{MB\_rot}\cdot\overline{Z}_{LOS} &= \cos\theta.
\end{aligned}
$$

Since the main beam polarization direction is aligned with the $\overline{X}_{MB}$ axis, the disalignment in the polarization induced by ξ can be found computing the angle, Ψ, between $\overline{X}_{MB}$ and $\overline{X}_{MB\_rot}$. Starting from the matrix representing the new rotated main beam frame with respect to the original one, the three Euler angles α, β, and γ can be reconstructed. Since they are related to the GRASP8 angles θ, φ, and ψ by $(\alpha,\ \beta,\ \gamma) = (\varphi,\ \theta,\ \psi-\varphi)$, the angle Ψ can be easly computed as $\gamma+\alpha$:

$$
\begin{aligned}
\Psi = &-\arctan\left[\frac{2\cos\theta\cos(\varphi-\psi)\sin(\frac{\xi}{2})^2-\sin\xi\sin(\varphi-\psi)}{\cos\varphi[\cos\psi\sin\xi+\cos\theta(-1+\cos\xi)\sin\psi]+\sin\varphi[-\cos\theta(-1+\cos\xi)\cos\psi+\sin\xi\sin\psi]}\right]+\\
&-\arctan\left[\frac{2\cos\theta\cos(\varphi-\psi)\sin(\frac{\xi}{2})^2+\sin\xi\sin(\varphi-\psi)}{\cos\varphi[\cos\psi\sin\xi-\cos\theta(-1+\cos\xi)\sin\psi]+\sin\varphi[-\cos\theta(-1+\cos\xi)\cos\psi+\sin\xi\sin\psi]}\right].
\end{aligned}
$$

The power series expansion for Ψ about $\theta = 0$ degrees (θ_{MB} are very small for the LFI main beams) to the first order, is independent from φ and ψ angles and it is equal to $-\xi$:

$$
\Psi_{\theta\to 0} = -\arctan\left[\tan\left(\frac{\xi}{2}+\varphi-\psi\right)\right]-\arctan\left[\tan\left(\frac{\xi}{2}-\varphi+\psi\right)\right]+O[\theta]^2.
$$