Internal Report IASF/CNR–BO 358/2002

MAIN BEAM SIMULATIONS FOR THE BEAST OPTICS

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November 2002

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Abstract

This technical note describes the work performed at IASF/CNR – Sezione di Bologna on the main beam simulations of the BEAST experiment. Simulations have been carried out using GRASP8 and considering perfect gaussian feed models and ideal reflectors.

1. Introduction

GRASP8 software has been used for simulating the response of the BEAST telescope. GO/GTD has been used on the sub–reflector while for the main reflector PO has been applied. Each beam has been calculated in a wide *(u,v)* regular grid between (–0.087, 0.087) both in *u* and *v* in the Line of Sight (LOS) reference frame in order to calculate the position of the beam peak. Once the peak location has been found, a convergency test has been performed and beams have been calculated in the peak centered coordinate system in a more refined grid of 301×301 points in the range (–0.02 , 0.02) both in *u* and *v*. All the calculation have been performed neglecting any focal plane and telescope symmetry. This means that, for example, the channel A and B have been calculated separately. As expected the results are in agreement with the symmetry of the Optics.

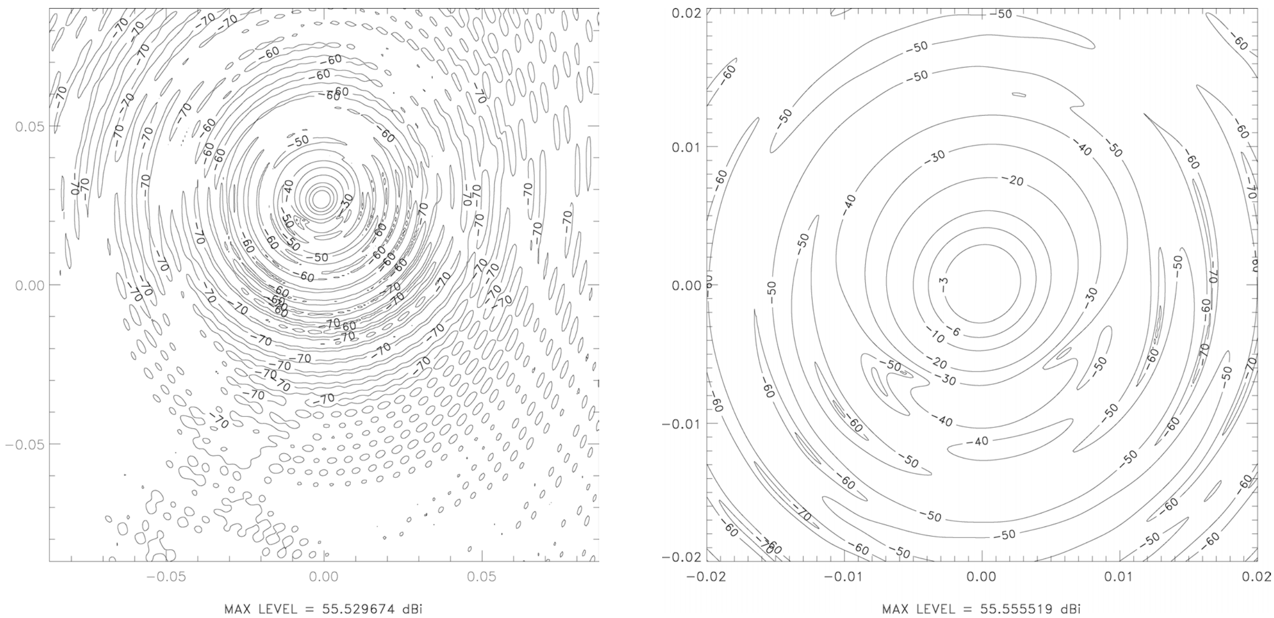


***Figure 1** – Example of the beam calculation (channel A). On the left the calculations have been performed in a wide range in the LOS system in order to find the beam peak location in (u,v) coordinates. Note the noise due to the poor sampling rate of the main reflector surface, for this angular region. On the right the same beam calculated after the convergency test and in a frame centred on the peak. For both plots the abscissa is the u coordinate and the vertical axis is the v coordinate.*

The model of the feed at both frequencies is Gaussian (see Table 1). The Gaussian model is an object in GRASP8 which permit to include near field effects in the calculations. However, these calculations have been performed neglecting any near field effect. In fact, since the feed aperture is $\sim 4.5\cdot\lambda$, the far field is at distance of $2\cdot D^2/\lambda = 2\cdot D^2\cdot\lambda \approx 400\,\text{mm}$ at 30 GHz. The typical distance between the feeds and the sub reflector is approximately 900 mm.

Channels	Model	Frequency [GHz]	Taper [dB]	Angle of Taper [°]
0, A, B, C, D, E	Gaussian	41.5	-13.89	20
F,G	Gaussian	30.0	-13.67	20

Table 1 *– Characteristics of the feed model used in GRASP8 for main beam calculations*

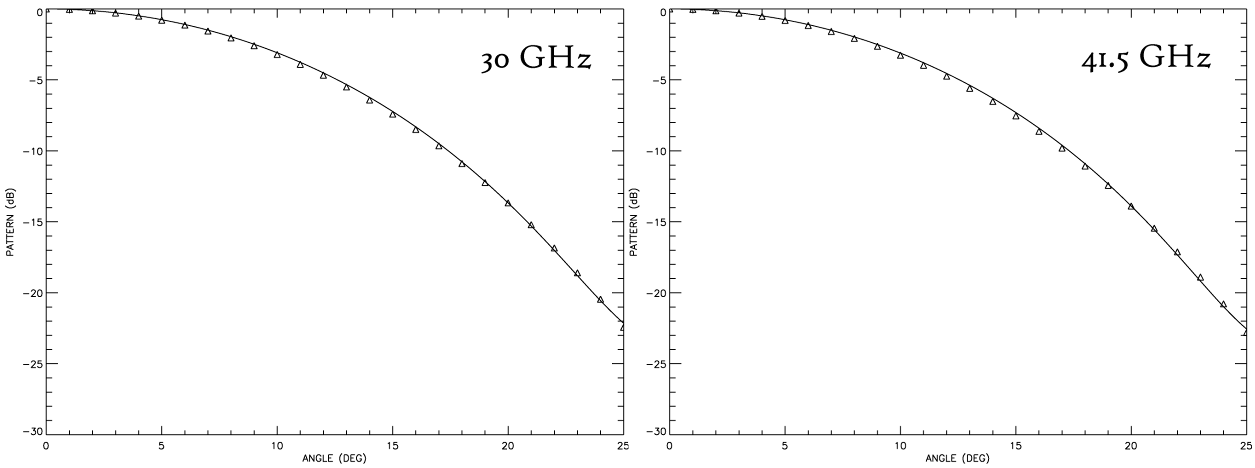


Figure 2 *– Comparison, up to 25 degree from the boresight direction, between the gaussian model (triangles) pattern of the feeds and the pattern simulated with modal coefficients at the aperture (solid line).*

2. Beam Peak

As previously mentioned, the beam peak location has been found. Then the parameters of the coordinate system centered at the peak as been carried out as reported in Table 2. The table reports also the location of the peak in *(u,v)* coordinate system as derived from the simulations. According with the GRASP8 formalism (see Figure 3), the three angles (θ,ϕ,ψ) have been reported in table.

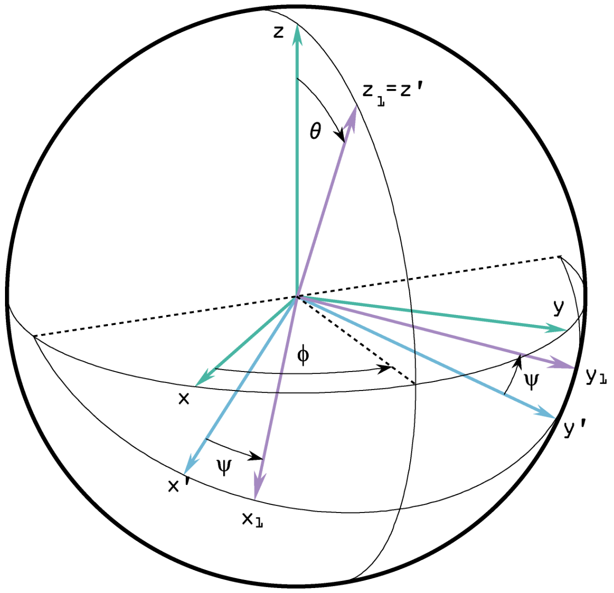


Figure 3 *– Coordinate system rotation as defined in GRASP8. Adapted form [1].*

MB ID	ν [GHz]	U	V	θ [°]	ϕ [°]	ψ [°]
O	41.5	0.00000	0.00000	0.000000	0.000000	0.000000
A	41.5	-0.00082	0.02677	1.534711	91.754496	0.000000
B	41.5	-0.00082	-0.02677	1.534711	-91.754496	0.000000
C	41.5	0.01936	0.01960	1.578664	45.352947	0.000000
D	41.5	0.01936	-0.01960	1.578664	-45.352947	0.000000
E	41.5	0.027890	0.000000	1.598187	0.000000	0.000000
F	30.0	-0.021940	0.013930	1.489206	147.587989	0.000000
G	30.0	-0.021940	-0.013930	1.489206	-147.587989	0.000000

Table 2 *Main beam peak location in (u,v) and (θ,ϕ,ψ).*

The angle ψ has not been applied since it defines the polarization alignment of the centered coordinate system.

3. Directivity, XPD, and Depolarization Parameter

The directivity of the copolar (D_p) and cross-polar (D_x) components have been calculated. Then the cross polar discrimination factor (XPD)[1] has been obtained:

$$\mathrm{XPD} = \frac{D_x}{D_p}$$

Since the reference frame in which the main beam has been calculated is not aligned properly with the electric field of the copolar component, the XPD is not the best value achievable. However, as readily seen in table, it remains 27 dB down to the copolar component.

The depolarization parameter has been obtained in the following way. The stokes parameters have been calculated for each beams, for each point in the regular *(u,v)* grid.

$$S_I(u,v) = E_{cp}(u,v)^2 + E_{xp}(u,v)^2$$

$$S_Q(u,v) = E_{cp}(u,v)^2 - E_{xp}(u,v)^2$$

$$S_U(u,v) = 2 \cdot E_{cp}(u,v) \cdot E_{xp}(u,v) \cdot \cos[\delta\varphi(u,v)]$$

$$S_V(u,v) = 2 \cdot E_{cp}(u,v) \cdot E_{xp}(u,v) \cdot \sin[\delta\varphi(u,v)]$$

in which $E_{co}(u,v)$ and $E_{xp}(u,v)$ are the amplitude field of the co–polar and cross–polar components respectively, $\delta\varphi$ is the phase difference between the co–polar and cross–polar fields. Then, over the whole *(u,v)* plane calculated each parameter has been summed.

$$S_N = \sum_{(u,v)} S_N(u,v) \cdot \Delta u \Delta v, N = I, Q, U, V$$

and finally

[1] this ratio is improperly called XPD. Actually the Cross – Polar discrimination is calculated punctually as the ratio between the cross–polar and the co–polar component. Then the XPD is a function of *(u,v)* coordinates. In this case the XPD is an indication of the maximum cross–polar level normalized to the co–polar peak.

$$\mathrm{Dep\%} = \left(1 - \frac{\sqrt{S_Q^{\ 2} + S_U^{\ 2} + S_V^{\ 2}}}{S_I}\right) \cdot 100$$

4. Gaussian Fit and Angular Resolution

Because of the beam aberrations, the angular resolution is function of the azimuthal cut of the beam (i.e. the beam, is not circular). Firstly the maximum and minimum FWHM have been calculated at using the GRASP8 post processor. Then the elliptical Gaussian fit of the beam in the (u,v) plane has been performed. The maximum, minimum and average FWHM has been carried out from the fit.
To perform the Gaussian fit of the beams an IDL routine "GAUSS2DFIT" has been applied [2]. The function fits a two-dimensional, elliptical Gaussian equation to rectilinearly gridded data as the *.grd file output is:

$$Z = f(x, y)$$

where:

$$f(x, y) = A_0 + A_1 \cdot e^{(-U/2)}$$

and the elliptical function is:

$$U = \left(\frac{x'}{a}\right)^2 + \left(\frac{y'}{b}\right)^2$$

the parameters of the ellipse U are:

- Axis lengths are $2a$ and $2b$, in the unrotated x and y axes, respectively.
- Rotation of *tilt* radians from the x axis, in the clockwise direction.

the rotated coordinate system is defined as:

$$x' = x \cdot \cos(tilt) - y \cdot \sin(tilt)$$
$$y' = x \cdot \sin(tilt) + y \cdot \cos(tilt)$$

MB ID	DIR [dBi]	XPD [dB]	FWHM max [']	FWHM min [']	FWHM ave [']	FWHMg X [']	FWHMg Y [']	FWHMg ave [']	Ell	Dep %	Tilt
O	55.70	-59.97	19.14	19.00	19.07	18.75	18.74	18.745	1.000	0.0004	0.00
A	55.56	-27.80	20.82	18.24	19.53	18.94	19.18	19.06	1.013	0.0897	-1.58
B	55.56	-27.80	20.56	18.24	19.70	18.94	19.18	19.06	1.013	0.0897	1.58
C	55.36	-31.59	21.23	18.70	19.97	20.27	18.68	19.475	1.085	0.0852	-25.56
D	55.36	-31.59	21.23	18.70	19.97	20.27	18.68	19.475	1.085	0.0852	25.56
E	55.28	-35.31	21.64	19.00	20.32	20.53	18.82	19.675	1.091	0.0806	0.00
F	53.04	-31.63	26.86	24.67	25.77	24.67	26.19	25.43	1.061	0.0870	14.61
G	53.04	-31.63	26.86	24.67	25.77	24.67	26.19	25.43	1.061	0.0870	-14.61

***Table 3** – Main beam characteristics. DIR=Directivity, XPD=cross – polar discrimination, FWHM (max,min,ave) = maximum, minimum and average beamwidth of the –3 dB contour. FWHMg (max,min,ave): maximum, minimum and average angular resolution of the fitted ellipse. Ell = ellipticity as derived from the fit. Dep=depolarization factor. Tilt=tilt angle of the fitted ellipse.*

The angular resolution as been calculated also from the contour area, Ω_b, at –3dB, using the GRASP postprocessor. The numbers have been reported in Table 4. For a circular beam, the area within the FWHM can be calculated as follows

$$\Omega_{\mathrm{FWHM}} = \int_0^{2\pi}\int_0^{\mathrm{FWHM}/2} \sin\theta \cdot d\theta \cdot d\phi = $$
$$= 2\pi \cdot \left[1 - \cos\frac{\mathrm{FWHM}}{2}\right]$$

Then

$$\mathrm{FWHM} = 2 \cdot \mathrm{acos}\left(1 - \frac{\Omega_{\mathrm{FWHM}}}{2\pi}\right)$$

If $\Omega_{\mathrm{FWHM}} = \Omega_b$ we can easily calculate the angular resolution of the symmetric beam with the same beam area.

MB ID	Area Contour -3dB [Sr]	Total Area [Sr]	FWHM [arcmin]
O	0.240E-04	0.160E-2	19.00
A	0.247E-04	0.160E-2	19.28
B	0.247E-04	0.160E-2	19.28
C	0.258E-04	0.160E-2	19.70
D	0.258E-04	0.160E-2	19.70
E	0.263E-04	0.160E-2	19.89
F	0.444E-04	0.160E-2	25.85
G	0.444E-04	0.160E-2	25.85

Table 4 – *Beam solid angle at –3dB and total solid angle.*

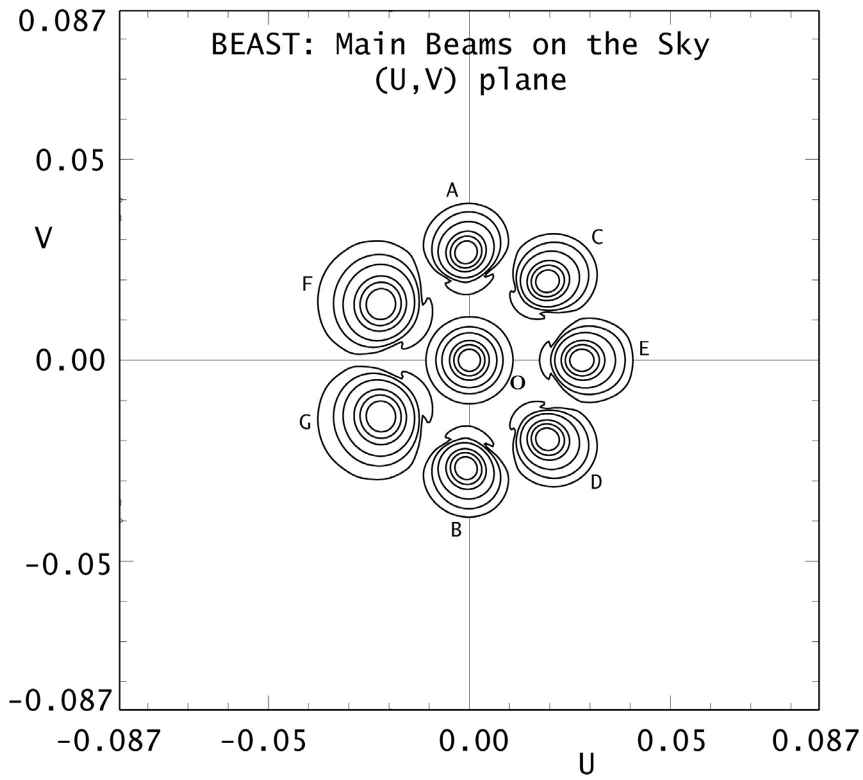


Figure 4 *– Main beam contour plots in the 10°x10° wide LOS centered (u,v) plane*

5. References

[1] GRASP8 Technical description

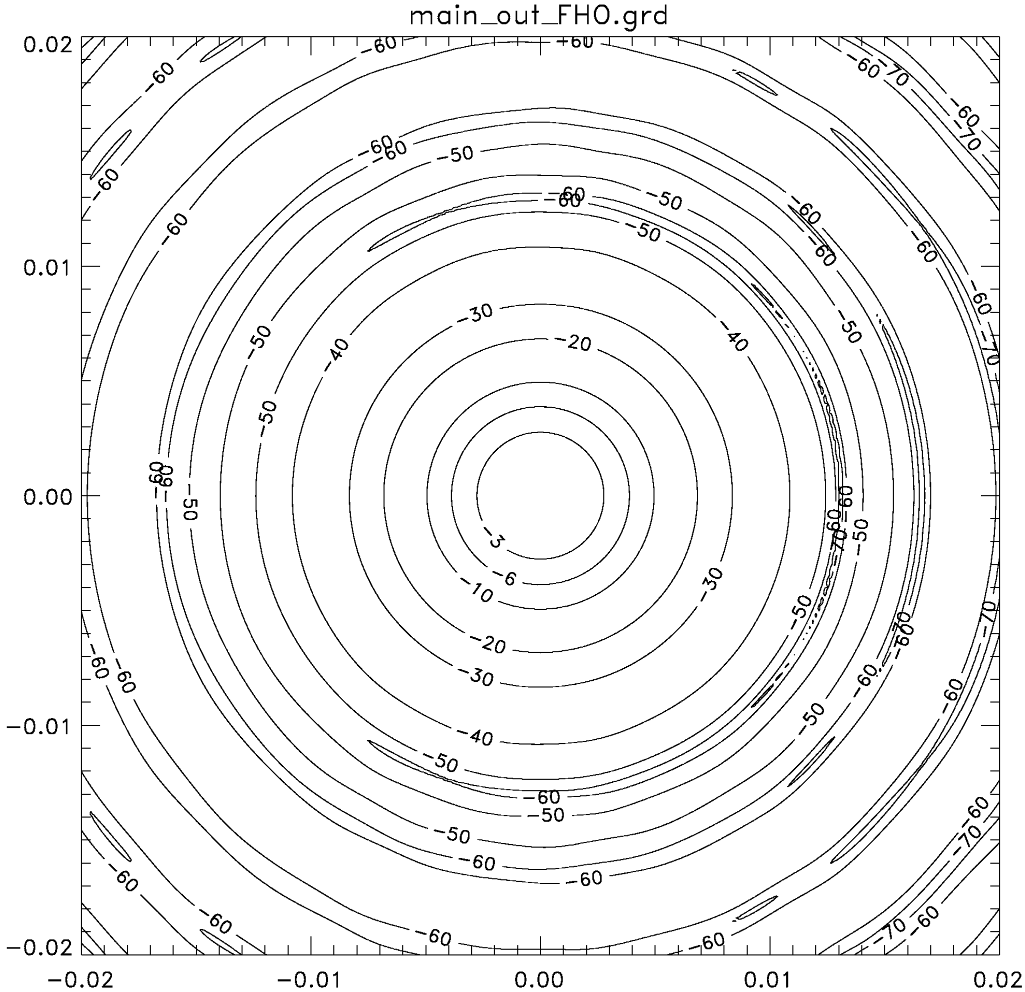
[2] IDL 5.4 user manual.

[3] F. Villa, M. Bersanelli, N. Mandolesi, *Design of Ka and Q band corrugated feed horns for CMB observations*, Internal Report ITESRE 188/1997

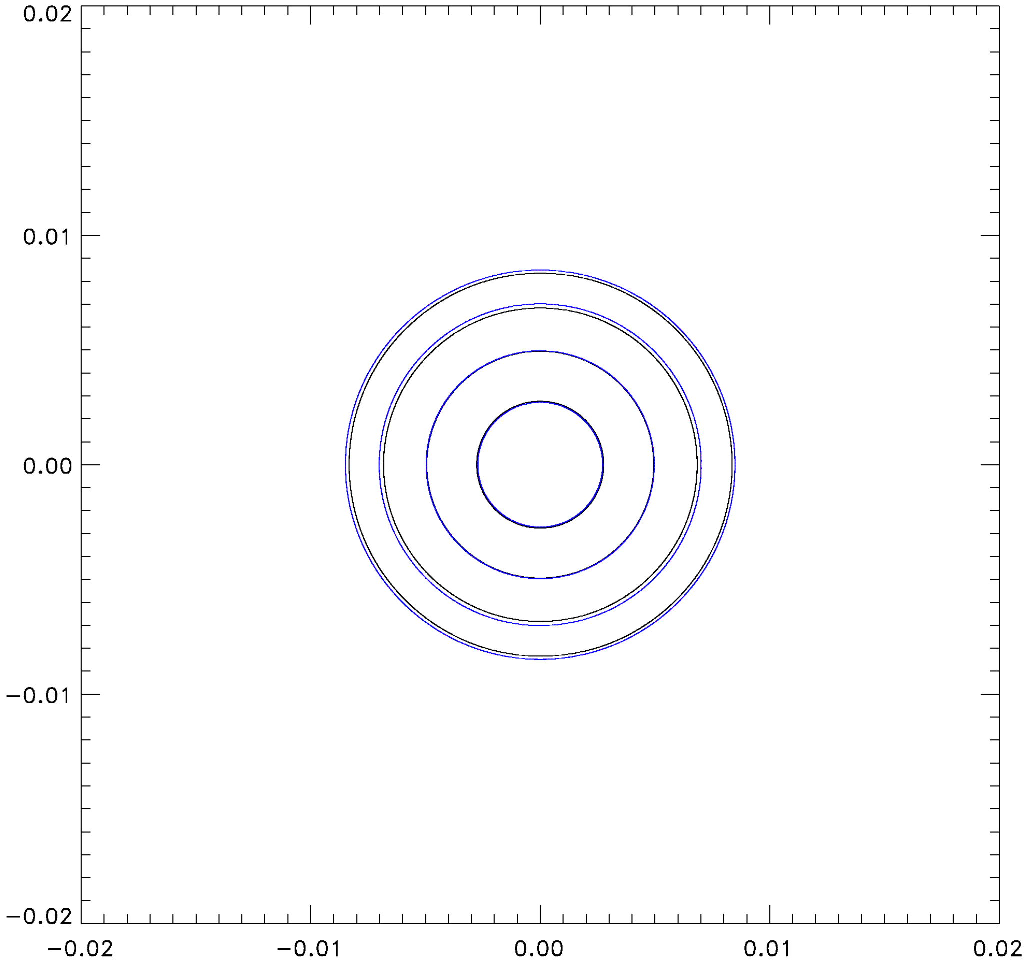
6. Plots

In the following pages the contour plot of the copolar component are shown for each beams. In addition, for each channel, the contour plot of the elliptical gaussian beam has been superimposed to the simulated beam (contour levels are –3, –10, –20, –30 dB from the peak).
The following parameters have been listed:

COMP1 MAX	Maximum level in dBi of the first component (co–polar in this case)
COMP2 MAX	Maximum level in dBi of the second component (cross–polar in this case)
XPD	Cross–polar discrimination in dB (see text)
% DEP	Percentage of the depolarization factor (see text)
FWHM X	Angular resolution along x of the fitted ellipse (arcmin)
FWHM Y	Angular resolution along y of the fitted ellipse (arcmin)
FWHM AVE	Average angular resolution
ELL	Ellipticity factor of the fitted ellipse.
TILT	Tilt of the fitted ellipse in degree.



MAX LEVEL = 55.699513 dBi



COMP1 MAX => 55.699513 dBi

COMP2 MAX => −4.2742083 dBi

XPD => 59.973721 dB

% DEPOL => 0.00027752396%

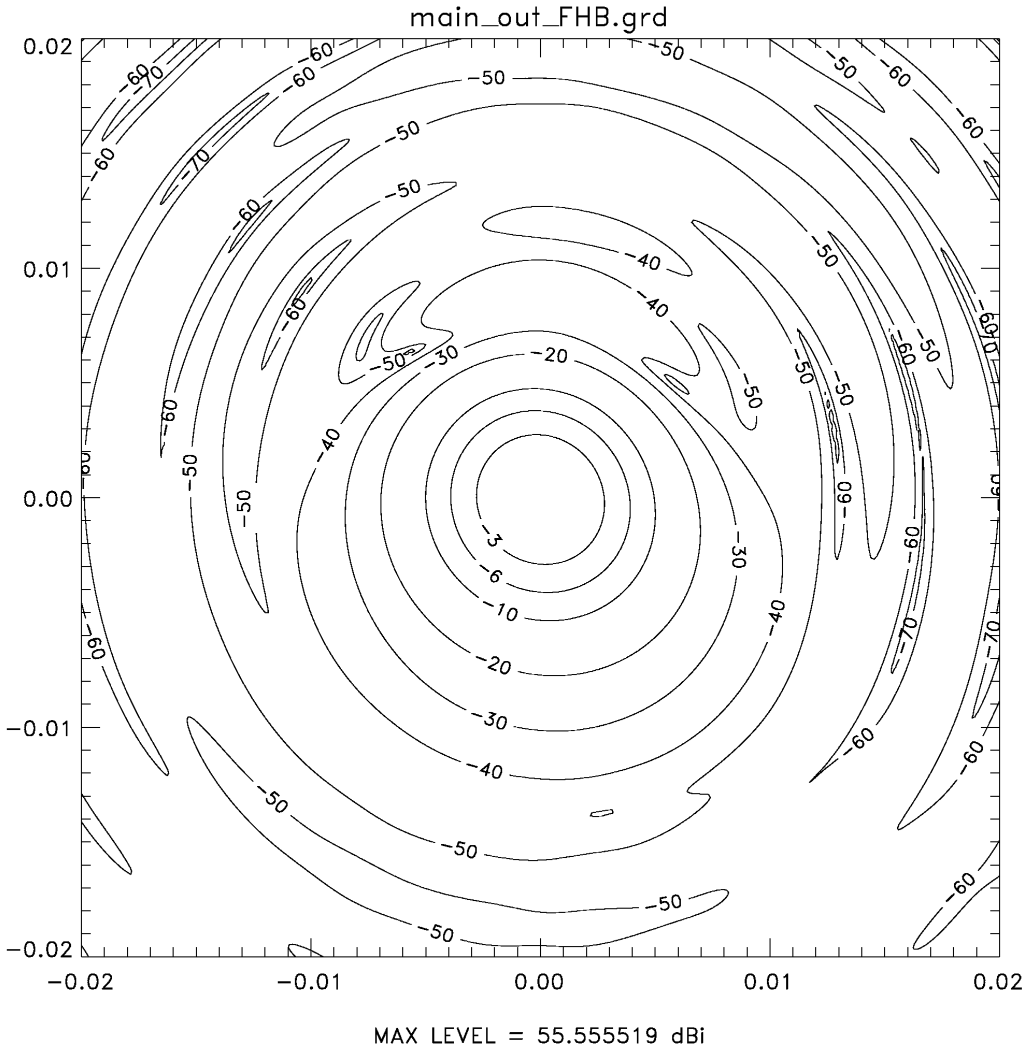
FWHM X => 18.752391 ARCMIN

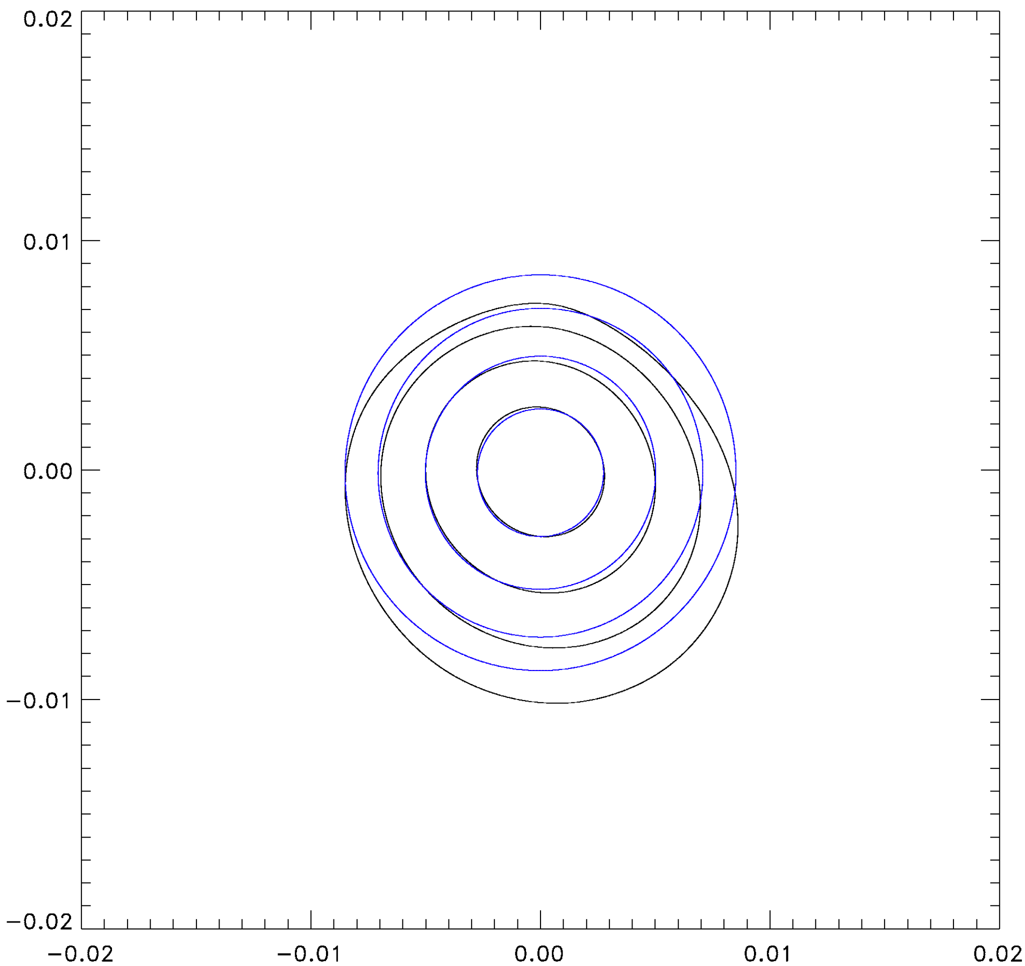
FWHM Y => 18.744680 ARCMIN

FWHM AVE => 18.748536 ARCMIN

ELL => 1.0004114

TILT => −2.9918174e−006 DEGREE


main_out_FHB.grd
0.02
0.01
0.00
−0.01
−0.02
−0.02
−0.01
0.00
0.01
0.02
MAX LEVEL = 55.555519 dBi



COMP1 MAX => 55.555519 dBi

COMP2 MAX => 27.760347 dBi

XPD => 27.795172 dB

% DEPOL => 0.089713333%

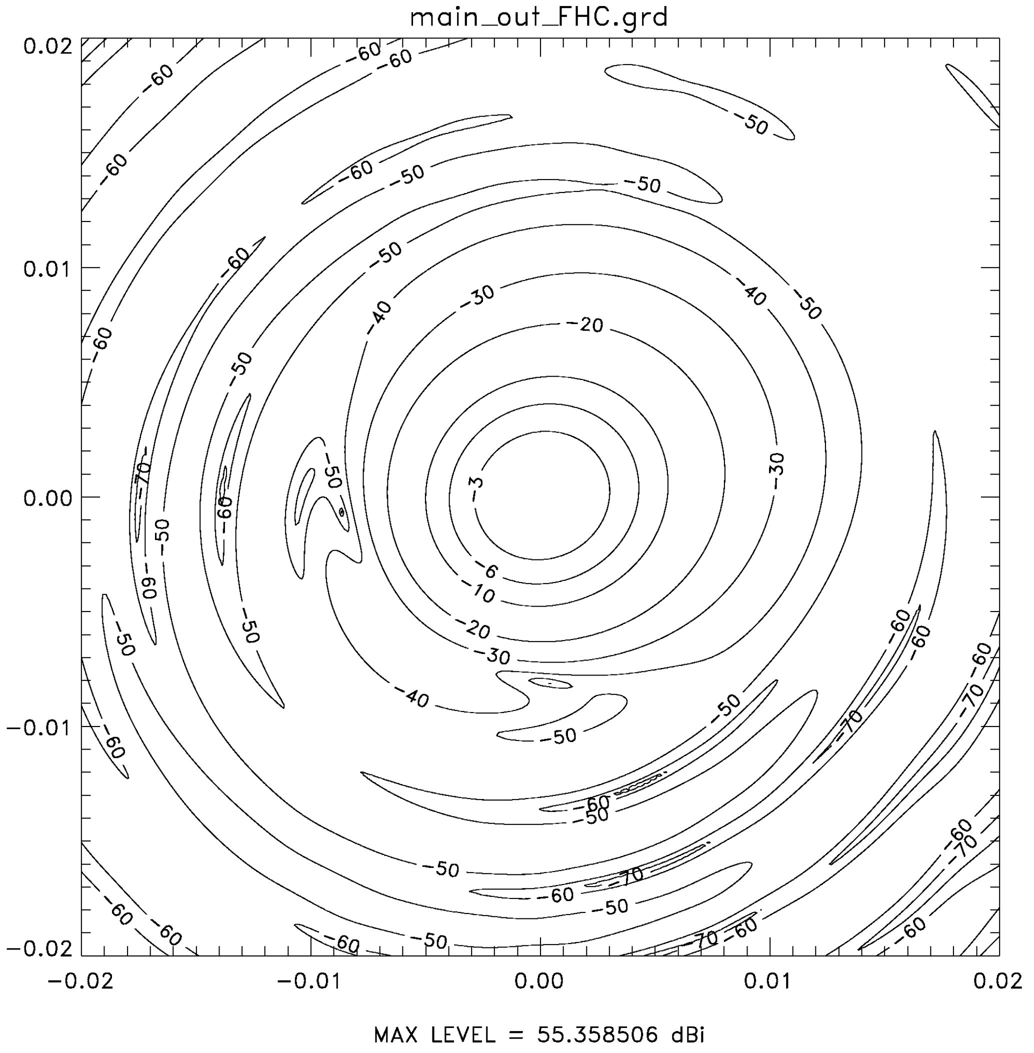
FWHM X => 18.939692 ARCMIN

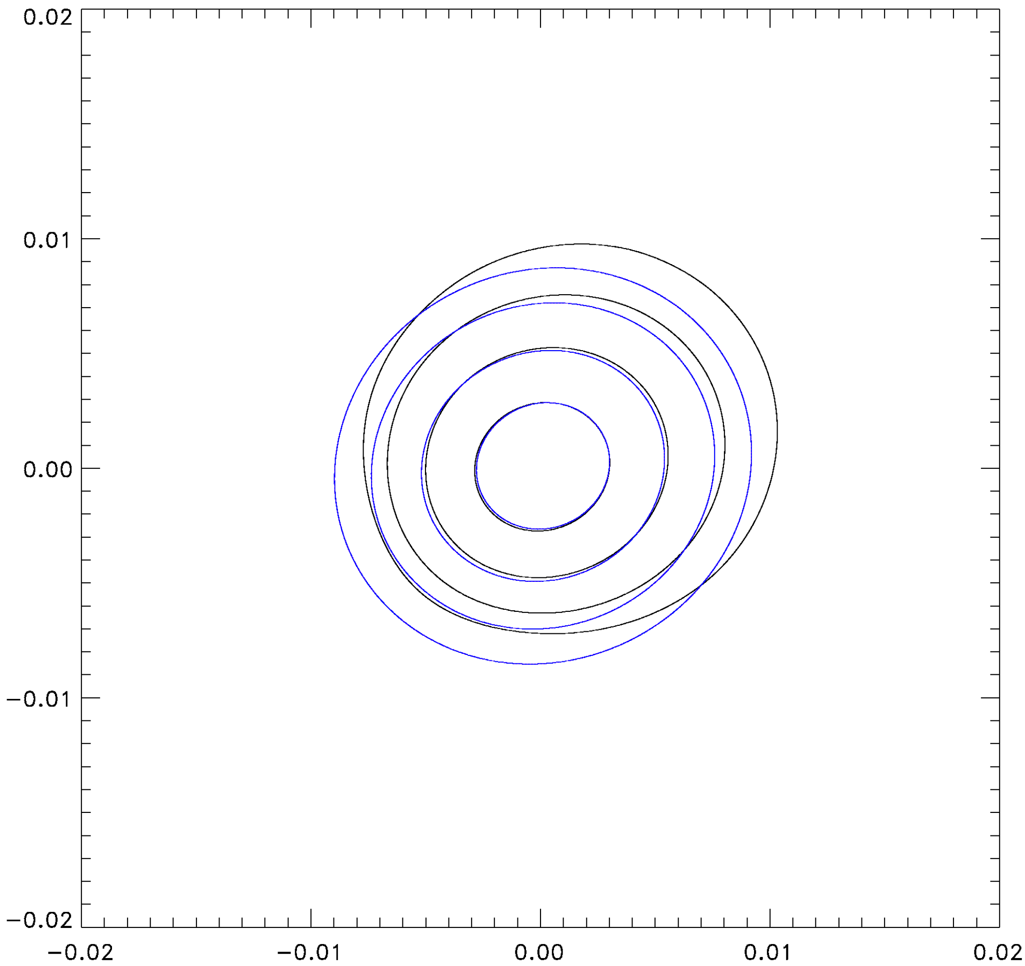
FWHM Y => 19.181774 ARCMIN

FWHM AVE => 19.060733 ARCMIN

ELL => 1.0127817

TILT => 1.5841971 DEGREE


main_out_FHC.grd
0.02
0.01
0.00
−0.01
−0.02
−0.02
−0.01
0.00
0.01
0.02
MAX LEVEL = 55.358506 dBi



COMP1 MAX => 55.358506 dBi

COMP2 MAX => 23.769054 dBi

XPD => 31.589452 dB

% DEPOL => 0.085158023%

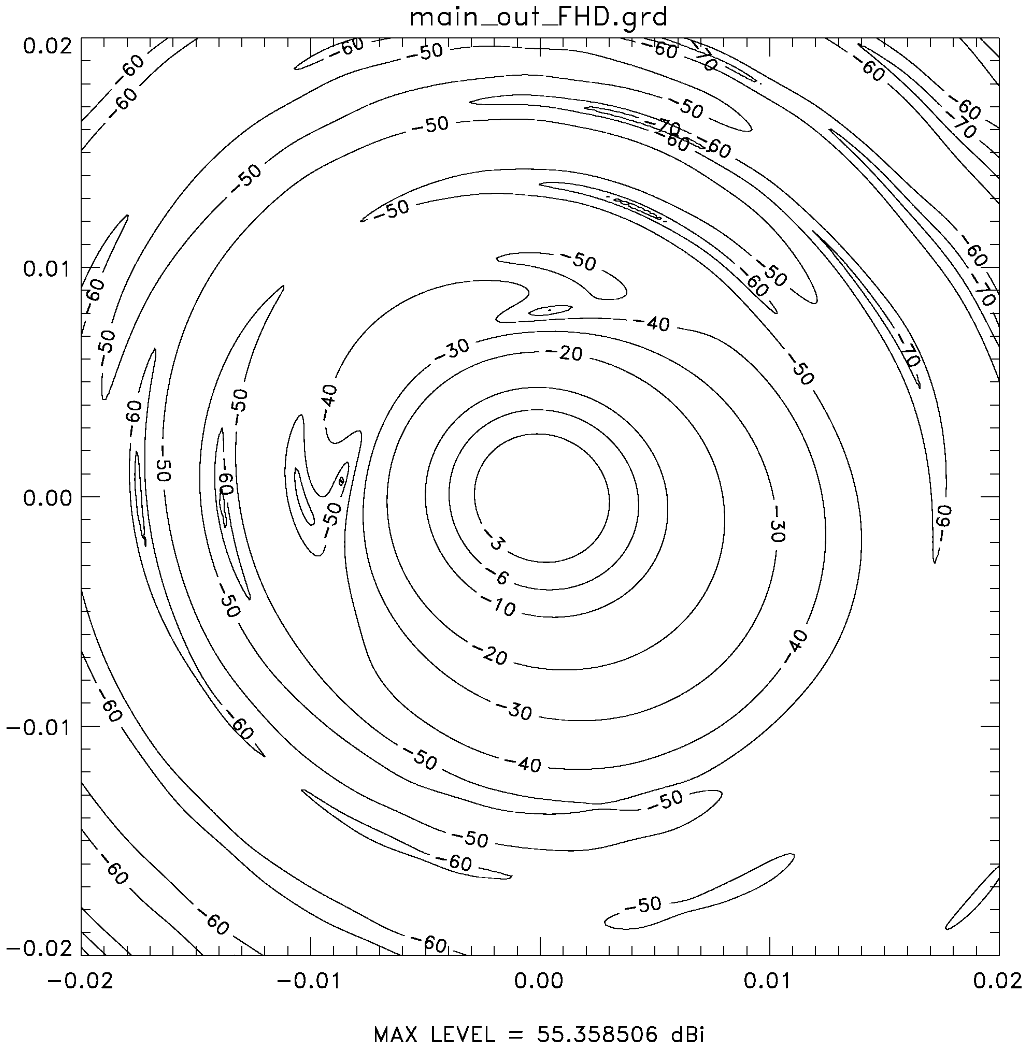
FWHM X => 20.271825 ARCMIN

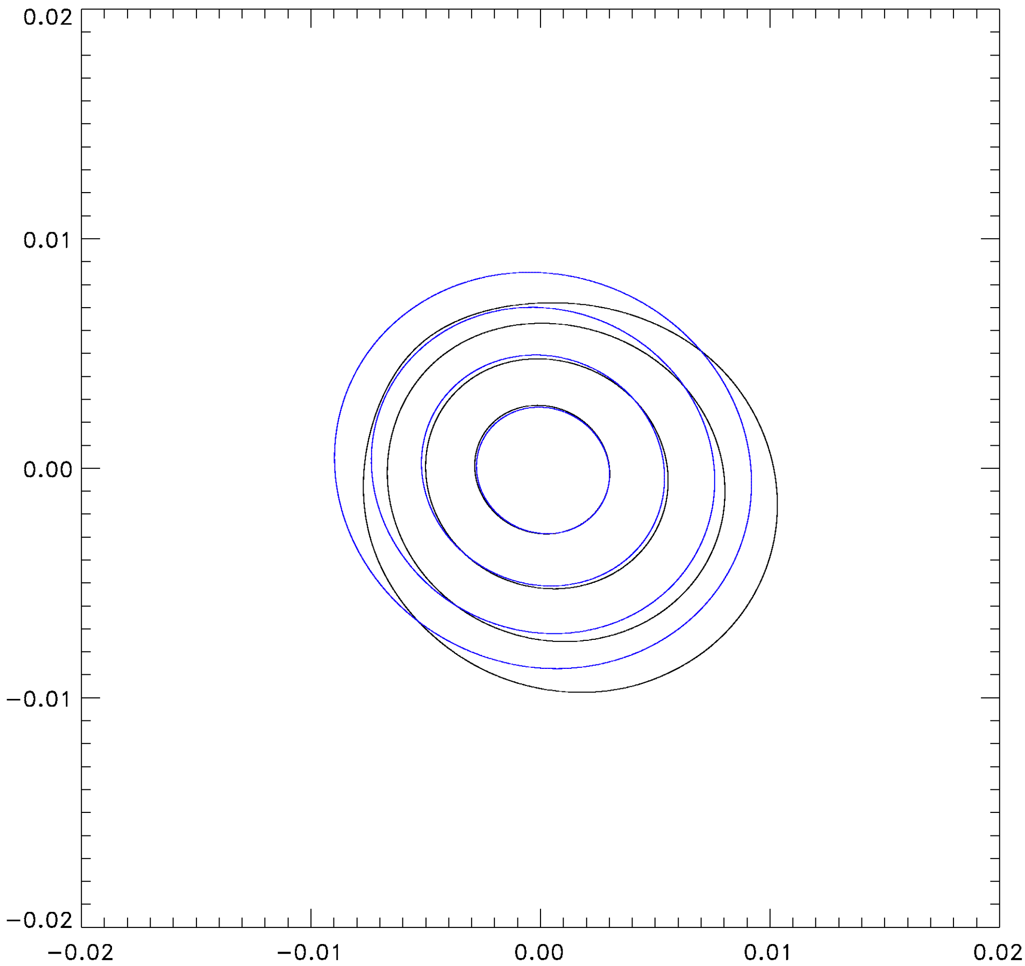
FWHM Y => 18.679792 ARCMIN

FWHM AVE => 19.475809 ARCMIN

ELL => 1.0852276

TILT => −25.559501 DEGREE


main_out_FHD.grd
0.02
0.01
0.00
−0.01
−0.02
−0.02
−0.01
0.00
0.01
0.02
−3
−6
−10
−20
−30
−40
−50
−60
−70
MAX LEVEL = 55.358506 dBi



COMP1 MAX => 55.358506 dBi

COMP2 MAX => 23.769054 dBi

XPD => 31.589452 dB

% DEPOL => 0.085158023%

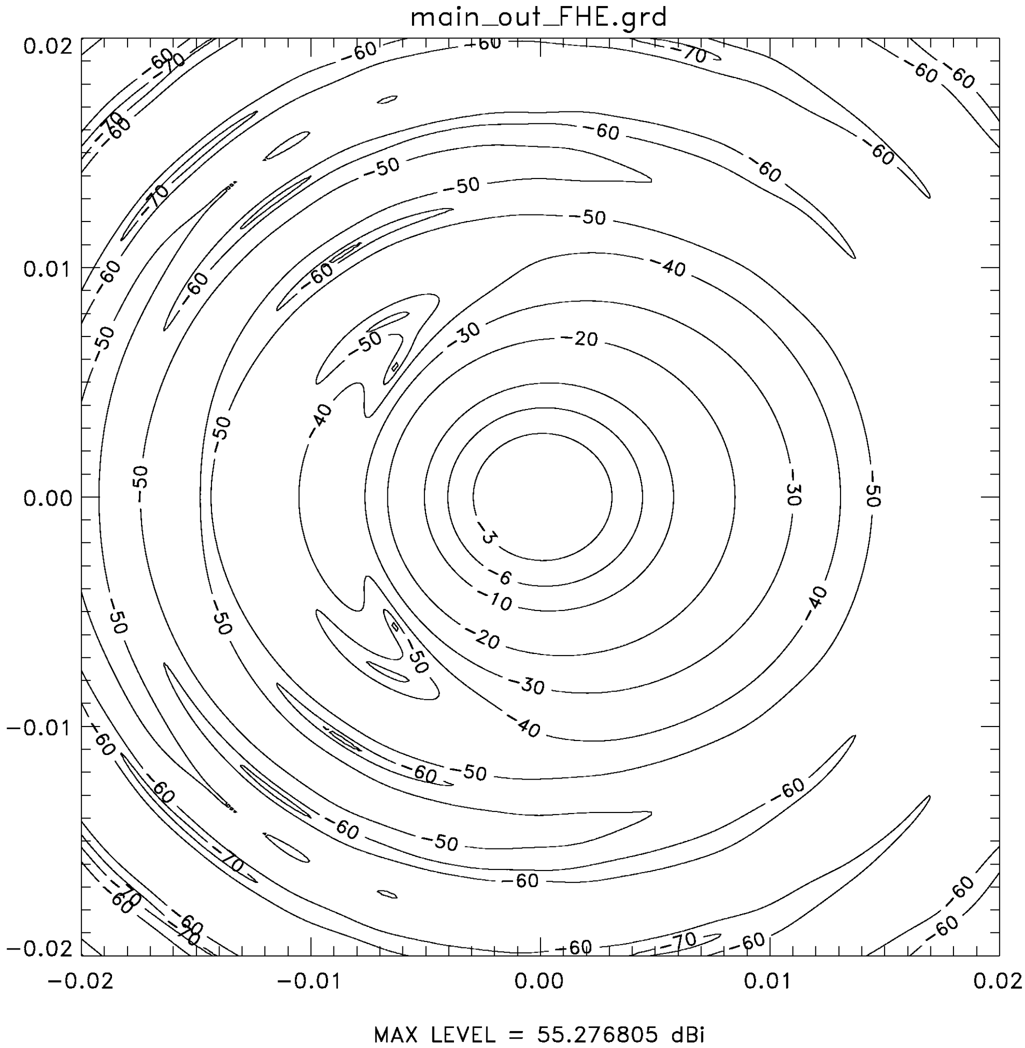
FWHM X => 20.271825 ARCMIN

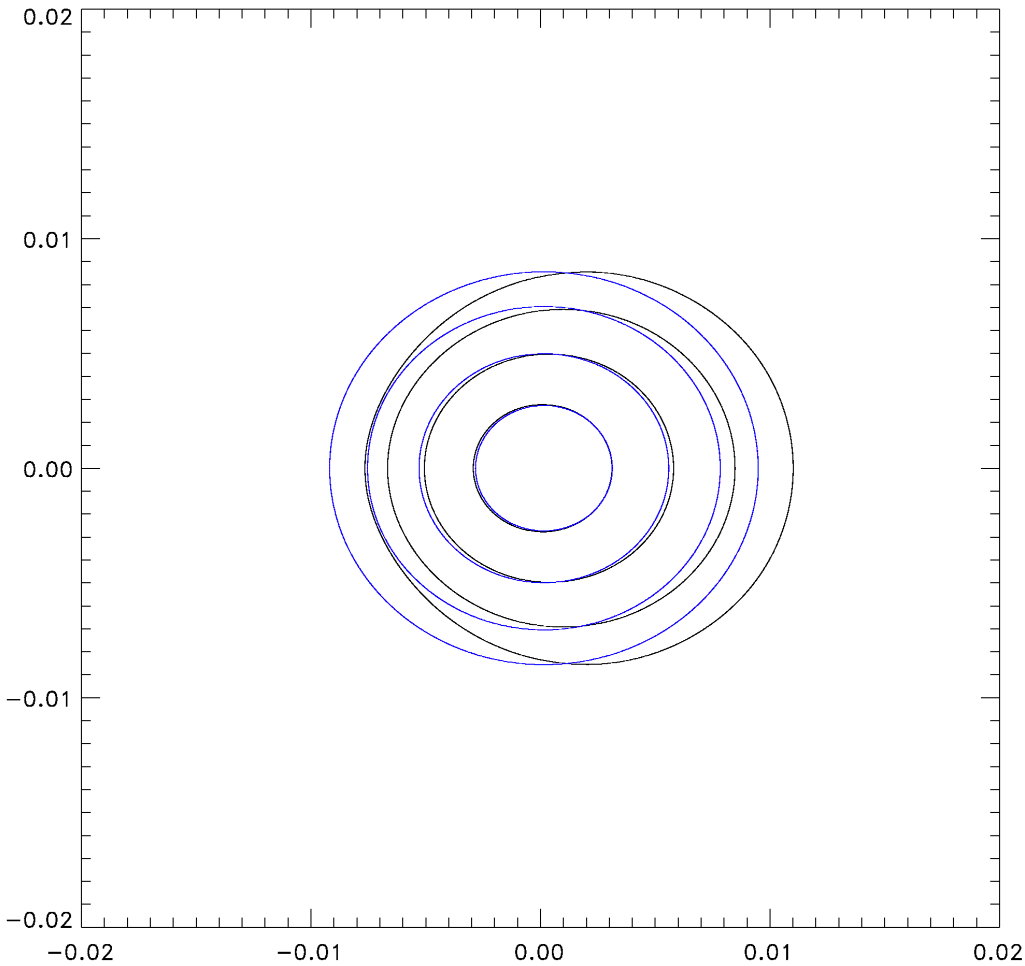
FWHM Y => 18.679792 ARCMIN

FWHM AVE => 19.475809 ARCMIN

ELL => 1.0852276

TILT => 25.559502 DEGREE


main_out_FHE.grd
0.02
0.01
0.00
−0.01
−0.02
−0.02
−0.01
0.00
0.01
0.02
−3
−6
−10
−20
−30
−40
−50
−60
−70
MAX LEVEL = 55.276805 dBi



COMP1 MAX => 55.276805 dBi

COMP2 MAX => 19.964751 dBi

XPD => 35.312054 dB

% DEPOL => 0.080585183%

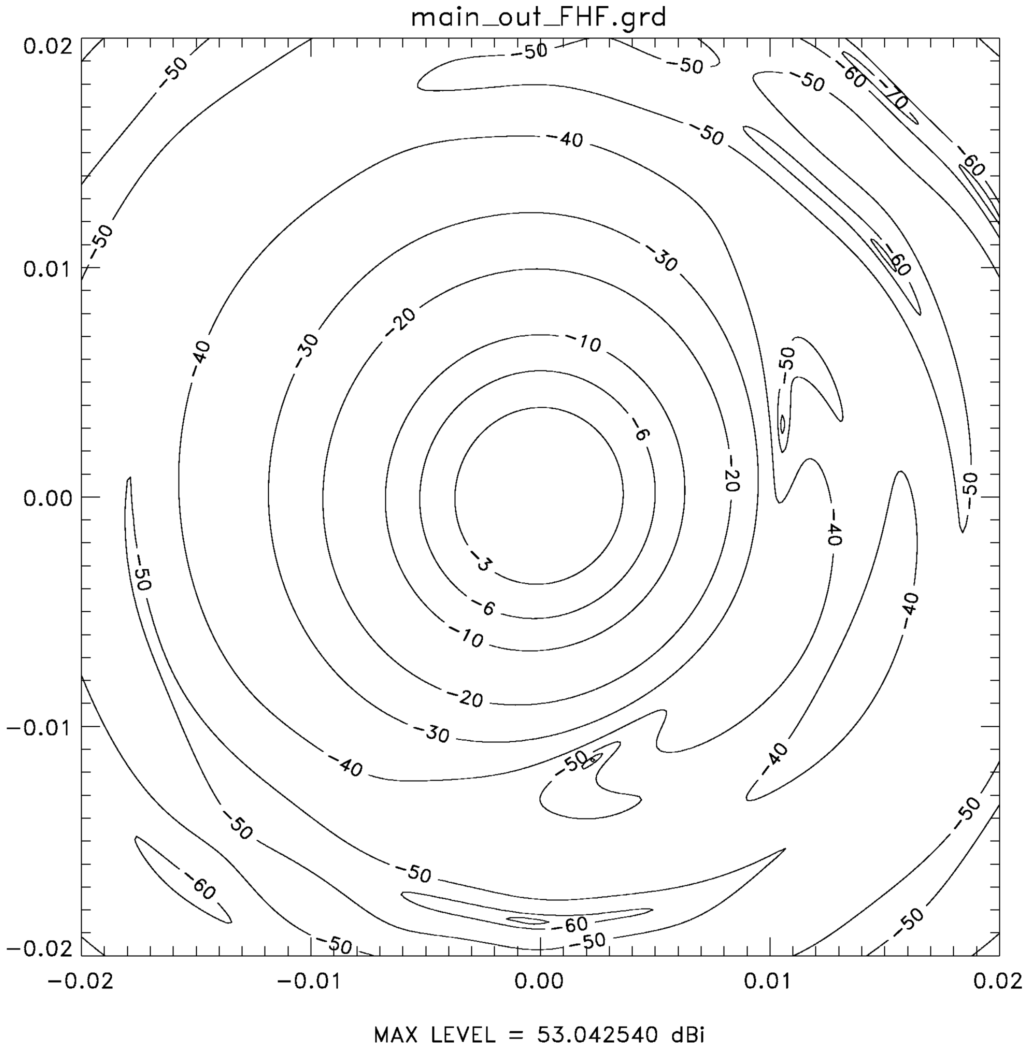
FWHM X => 20.530492 ARCMIN

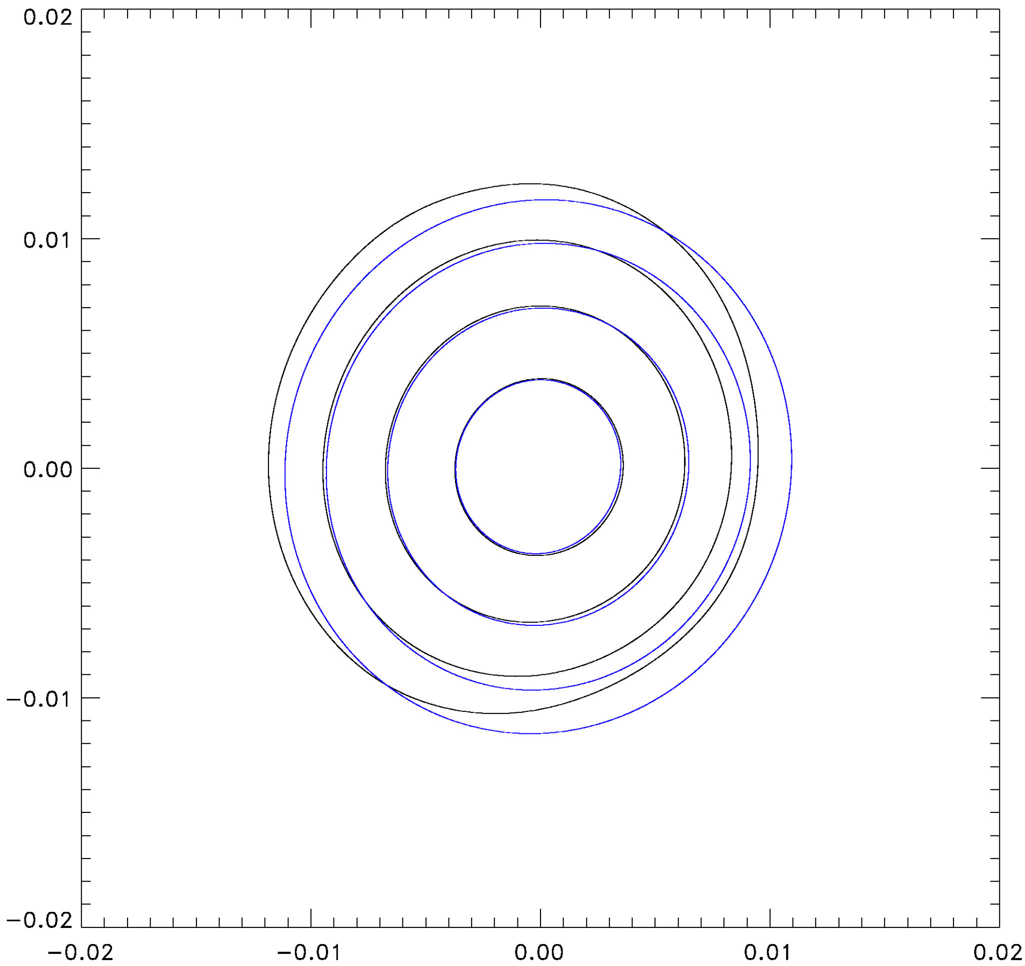
FWHM Y => 18.816401 ARCMIN

FWHM AVE => 19.673447 ARCMIN

ELL => 1.0910956

TILT => −6.0159235e−008 DEGREE


main_out_FHF.grd
0.02
0.01
0.00
−0.01
−0.02
−0.02
−0.01
0.00
0.01
0.02
−3
−6
−10
−20
−30
−40
−50
−60
−70
MAX LEVEL = 53.042540 dBi



COMP1 MAX => 53.042540 dBi

COMP2 MAX => 21.414302 dBi

XPD => 31.628238 dB

% DEPOL => 0.087043841%

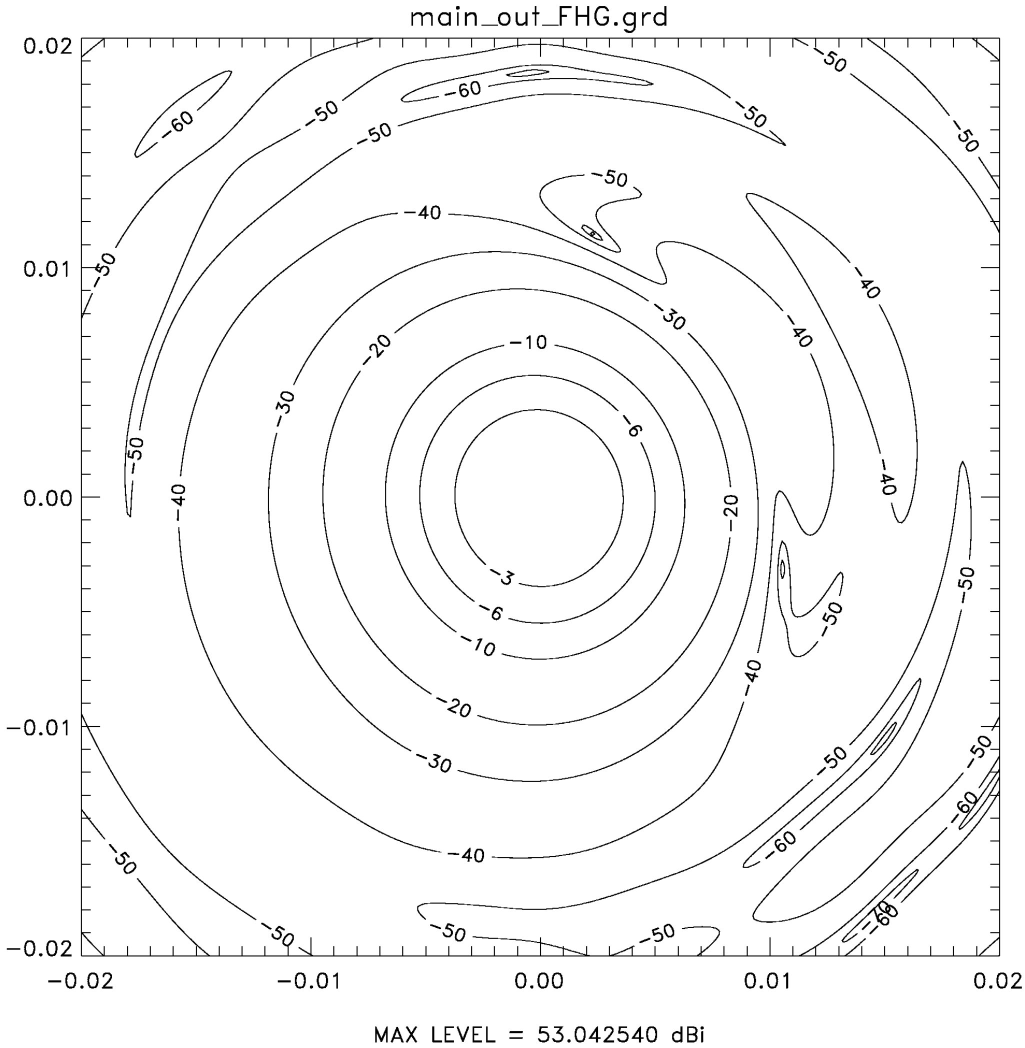
FWHM X => 24.674318 ARCMIN

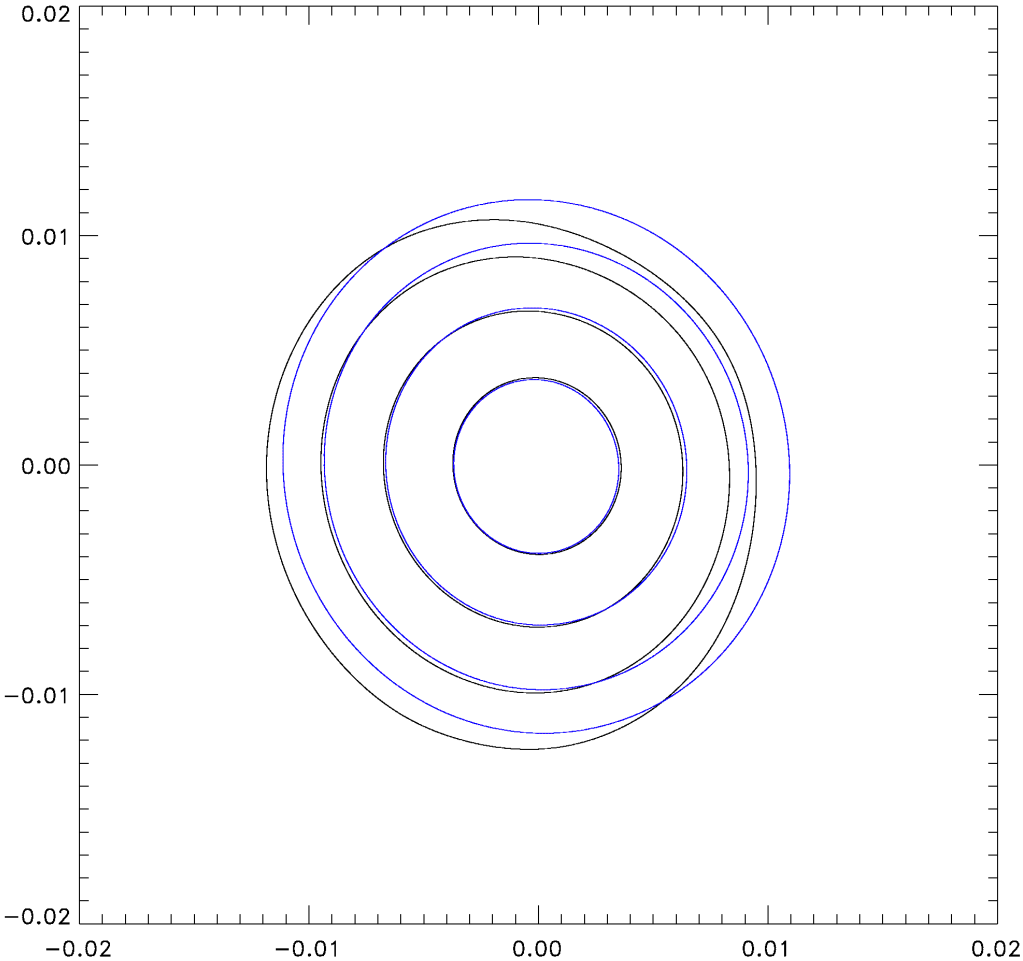
FWHM Y => 26.189534 ARCMIN

FWHM AVE => 25.431926 ARCMIN

ELL => 1.0614086

TILT => 14.606368 DEGREE


main_out_FHG.grd
0.02
0.01
0.00
−0.01
−0.02
−0.02
−0.01
0.00
0.01
0.02
−3
−6
−10
−20
−30
−40
−50
−60
MAX LEVEL = 53.042540 dBi



COMP1 MAX => 53.042540 dBi

COMP2 MAX => 21.414302 dBi

XPD => 31.628238 dB

% DEPOL => 0.087043841%

FWHM X => 24.674318 ARCMIN

FWHM Y => 26.189534 ARCMIN

FWHM AVE => 25.431926 ARCMIN

ELL => 1.0614086

TILT => −14.606367 DEGREE