Internal Report IASF 362/2002

BEAST OPTICS IN GRASP8

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February 2003

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SUMMARY — This technical Note reports the GRASP8 model of the BEAST optics. The Background Emission Anisotropy Scanning Telescope (BEAST)[1] is a 2 m diameter carbon fiber composite primary mirror for high angular resolution and a sensitive array of ultra-low noise HEMT amplifiers at 30, 40, and 90 GHz. It has been installed at the White Mountain Research Station Barcroft facility in 2000 and started the observations in July 2001. GRASP8 is a software developed by TICRA[2] for advanced reflector antenna analysis.

1 The BEAST Optics

The BEAST optical design is based on a two mirror off–axis scheme which offers the advantage of an unblocked aperture. A third off–axis flat mirror is located in front of the primary and it is used as wobbling system. The optical scheme is shown in Figure 1. The primary reflector is parabolic and the secondary one is elliptical as in conventional Gregorian configurations. The axis of the two reflectors are tilted at $2\beta = 35.41°$ in order to reduce the aberration of the off–axis scheme. The chief ray at the focal plane is tilted at $2\alpha = 58.24°$ with respect to the secondary mirror axis. The primary mirror aperture is 1966.1 mm and the aperture center is at 1255.2 mm from the parabola vertex. The primary mirror focal length is 1250 mm. The secondary mirror is elliptical with the major half axis of 866.68 mm and the minor half axis of 853.4 mm. The primary focus of the ellipse is located at the parabola focus. The Center of the Focal Plane unit is located at the secondary Focus of the ellipse.

[1] www.wmrs.edu/Research%20Programs/millimeter_wave_cosmology.htm

[2] TICRA Engineering Consultants – www.ticra.com

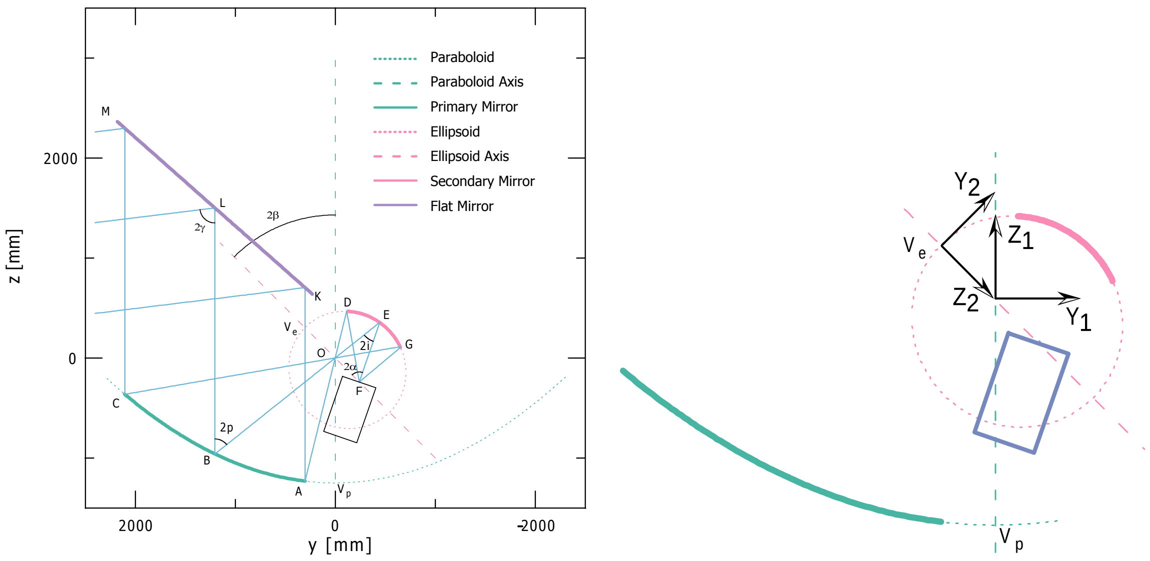

Figure 1: On the left, the BEAST Optics as shown in the coordinate system (X_1, Y_1, Z_1) (see text). On the right, a closed view with the two original coordinate systems. Adapted from [1]

2 Primary Mirror Definition

The primary mirror is defined by the following equations given in the coordinate system (X_1, Y_1, Z_1):

$$\begin{cases} Z_1 = \frac{1}{5000} \cdot (X_1^2 + Y_1^2) - 1250 \\ Z_1 = 0.50208 \cdot Y_1 - 1371.83 \end{cases} \tag{1}$$

The rim can be defined as the intersection between the paraboloid and a cylinder with a diameter of 1966.1 mm and the axis which pass trough the point $(0, -1255.2, 0)$.

It is useful to define the following coordinate system (`system_M1`) centered at the parabola vertex and used as a global coordinate system for GRASP8:

$$\begin{aligned} X_{\mathrm{M1}} &= -Y_1 \\ Y_{\mathrm{M1}} &= -X_1 \\ Z_{\mathrm{M1}} &= +Z_1 + 1250 \end{aligned} \tag{2}$$

In this case the Parabola equation is then

$$Z_{\mathrm{M1}} = \frac{1}{4 \cdot F_p} \cdot \left(X_{\mathrm{M1}}^2 + Y_{\mathrm{M1}}^2 \right), F_p = 1250mm \tag{3}$$

the center of the rim, projected in the $(X_{\mathrm{M1}}, Y_{\mathrm{M1}})$ plane, is at the point $(1255.2, 0, 0)$. The center of the rim at the reflector surface occurs at $(1255.2, 0, 315.105)$. The rim is circular and the radius is $R_p = 983.05$ mm. The following objects have been defined in GRASP8 (see [2] for details):

```
Main_Surface                      paraboloid
(
  focal_length     :     1250.000000      mm
```

```
)

Main_rim                          elliptical_rim
(
  centre           : struct(x:    1255.200000     mm,y:    0.000000000     mm),
  half_axis        : struct(x:    983.0500000     mm,y:    983.0500000     mm)
)

Main_reflector                    reflector
(
  coor_sys         : ref(system_M1),
  surface          : ref(Main_Surface),
  rim              : ref(Main_rim),
  centre_hole_radius               :    0.000000000     mm
)
```

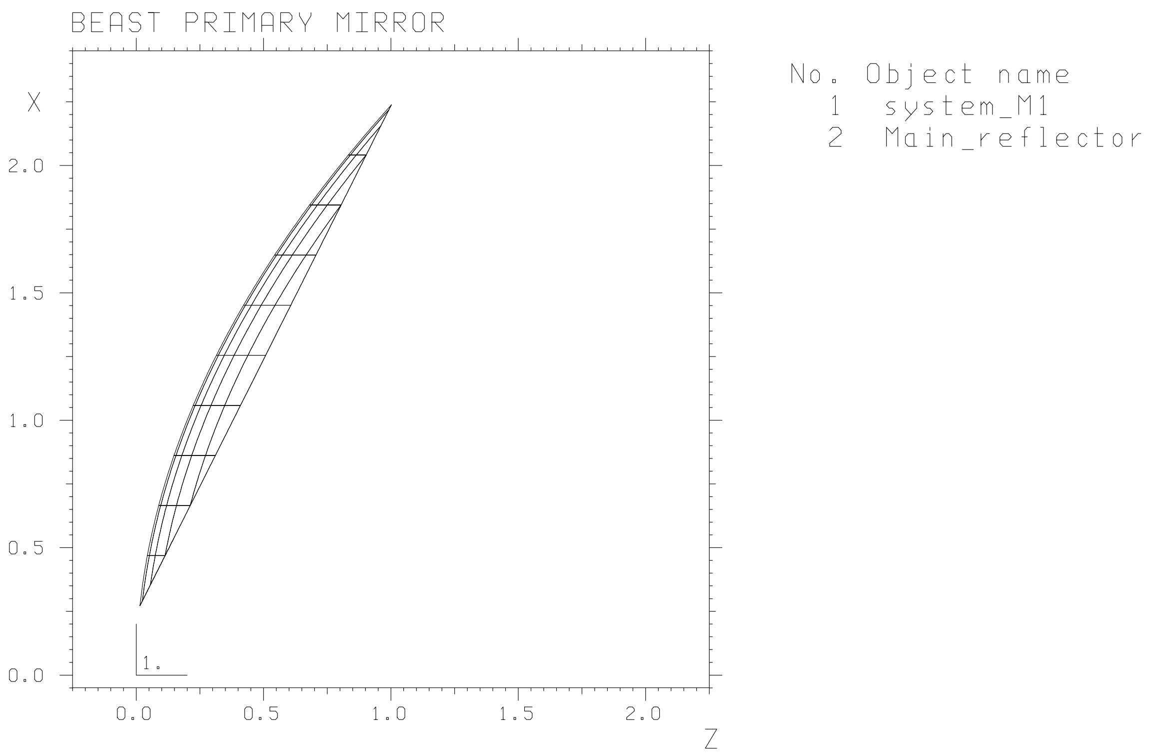

Figure 2: The Primary mirror as seen form the `system_M1` coordinate frame in the telescope symmetry plane.

3 Secondary Mirror Definition

In the coordinate system (X_2, Y_2, Z_2), the secondary mirror is defined by:

$$\begin{cases} \left(\frac{X_2}{b}\right)^2 + \left(\frac{Y_2}{b}\right)^2 + \left(\frac{Z_2-a}{a}\right)^2 = 1 \\ Z_2 = 2.96976 \cdot Y_2 - 1349.71 \end{cases} \tag{4}$$

The reflector must be defined in a coordinate system centered at one focus of the ellipse and with the x–axis or alternatively the y–axis aligned with the line which defines the rim in the symmetry plane of the telescope. It turn out that the angle between this line and the Y_2 axis is

$$\theta_{rim} = \tan^{-1}(2.96976) = 71.3902^\circ \tag{5}$$

The following coordinate systems have been defined. Firstly a `system_focal_point` frame has been setup with the origin at the parabola focus, the Z–axis pointing toward the origin of the `system_M1` and the X–axis parallel to the X–axis of the `system_M1`.

```
system_Focal_point                coor_sys
(
  origin : struct(x: 0.000000000 mm,y: 0.000000000 mm,z: 1250.000000 mm),
  y_axis : struct(x: 0.000000000   ,y: -1.000000000  ,z: 0.000000000    ),
  base   : ref(system_M1)
)
```

Then the `system_M2` has been defined starting from `system_focal_point` having the same origin and rotated in anticlock direction of 35.41°.

```
system_M2                         coor_sys
(
  origin : struct(x: 0.000000000 mm   ,y: 0.000000000 mm   ,z: 0.000000000 mm    ),
  x_axis : struct(x: 0.8150266797     ,y: 0.2265195034E-16,z: 0.5794234302      ),
  y_axis : struct(x: 0.2265195034E-16,y: 1.000000000      ,z: -0.7095656141E-16),
  base   : ref(system_Focal_point)
)
```

Finally the `system_M2_rim` has been defined:

```
system_M2_rim                     coor_sys
(
  origin : struct(x: 0.000000000 mm,y: 0.000000000 mm,z: 0.000000000 mm ),
  x_axis : struct(x: -0.3191214105 ,y: 0.000000000   ,z: 0.9477138415   ),
  y_axis : struct(x: 0.000000000   ,y: -1.000000000  ,z: 0.000000000    ),
  base   : ref(system_M2)
)
```

In the `system_M2_rim` coordinate system the rim has the equation given by the intersection from the ellipsoid

$$\begin{cases} \left(\frac{X_{\mathrm{M2\_rim}}}{b}\right)^2 + \left(\frac{Y_{\mathrm{M2\_rim}}}{b}\right)^2 + \left(\frac{Z_{\mathrm{M2\_rim}}-c}{a}\right)^2 = 1 \\ Z_{\mathrm{M2\_rim}} = -636.885 \end{cases} \tag{6}$$

It turn out that the rim is elliptical in this plane with equation given by:

$$\left(\frac{X_{\mathrm{M2\_rim}}}{472.275}\right)^2 + \left(\frac{Y_{\mathrm{M2\_rim}}}{448.718}\right)^2 = 1 \tag{7}$$

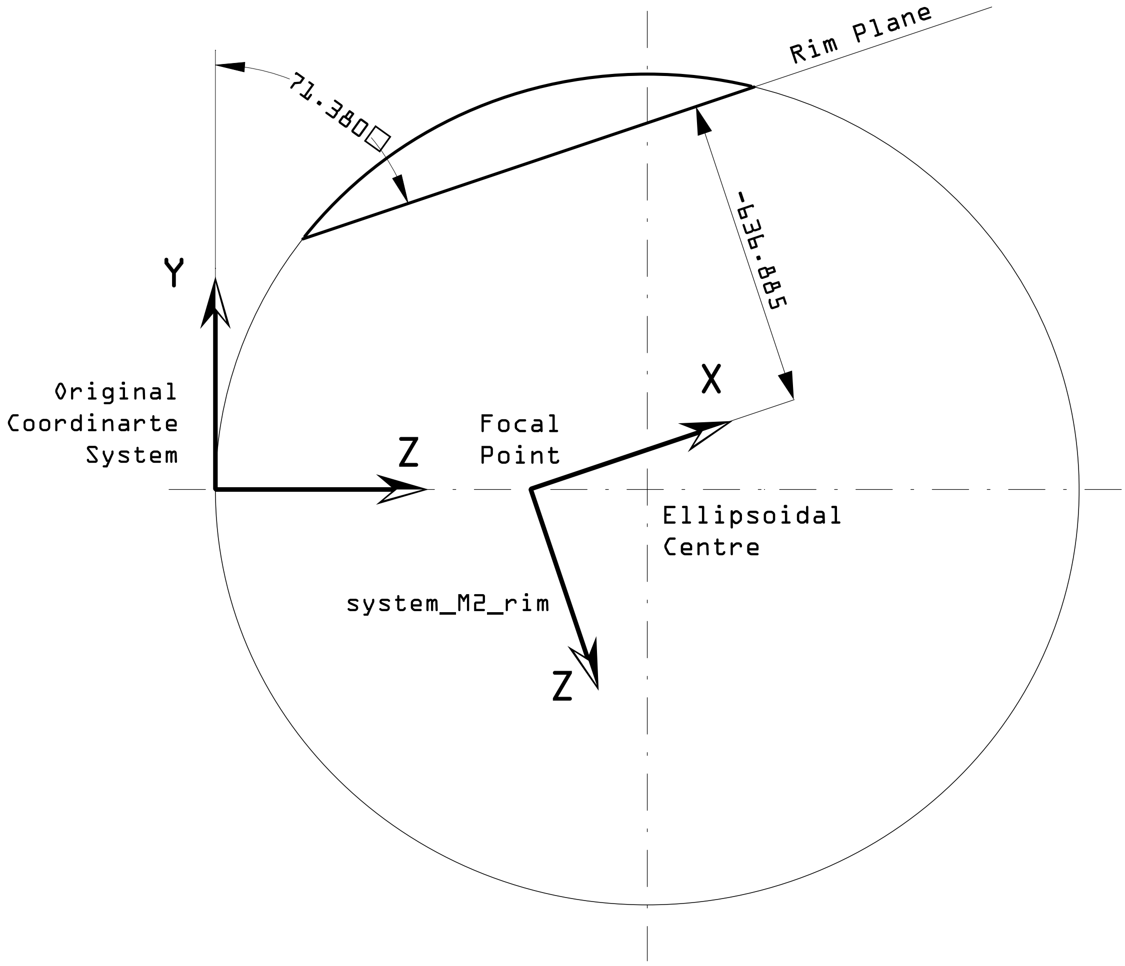

Figure 3: Secondary mirror and rim definition as seen from its center

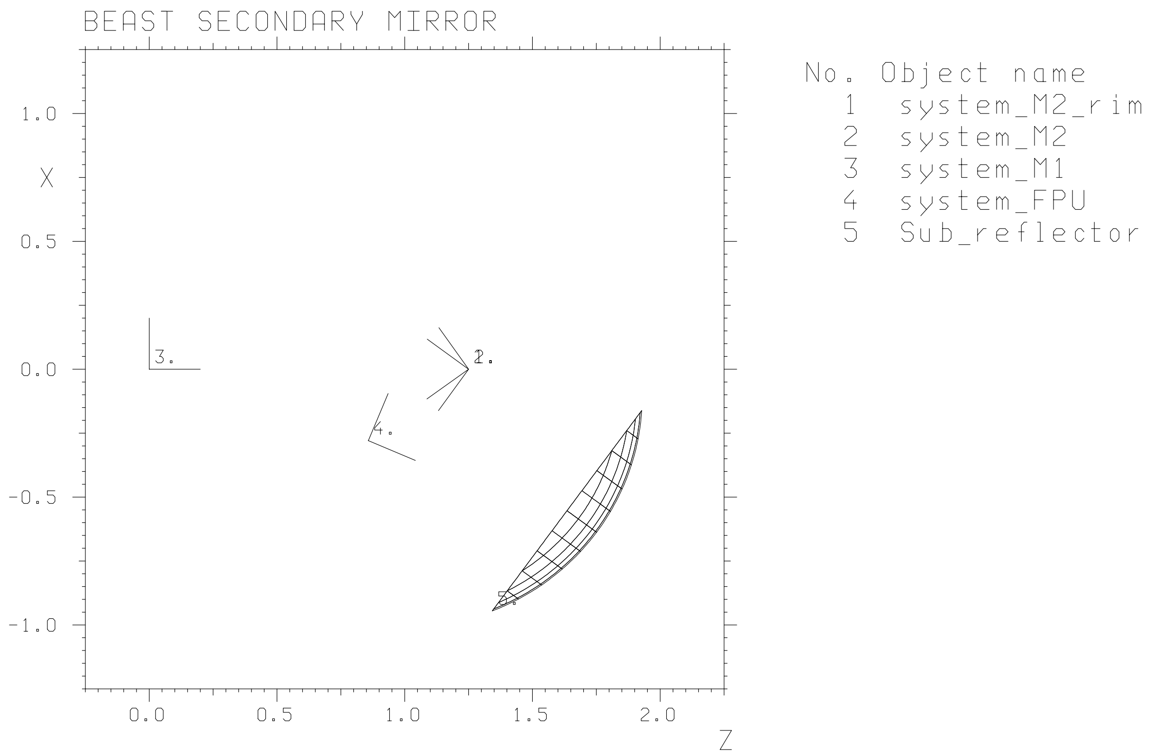

Figure 4: Secondary Mirror as seen from `system_M1`

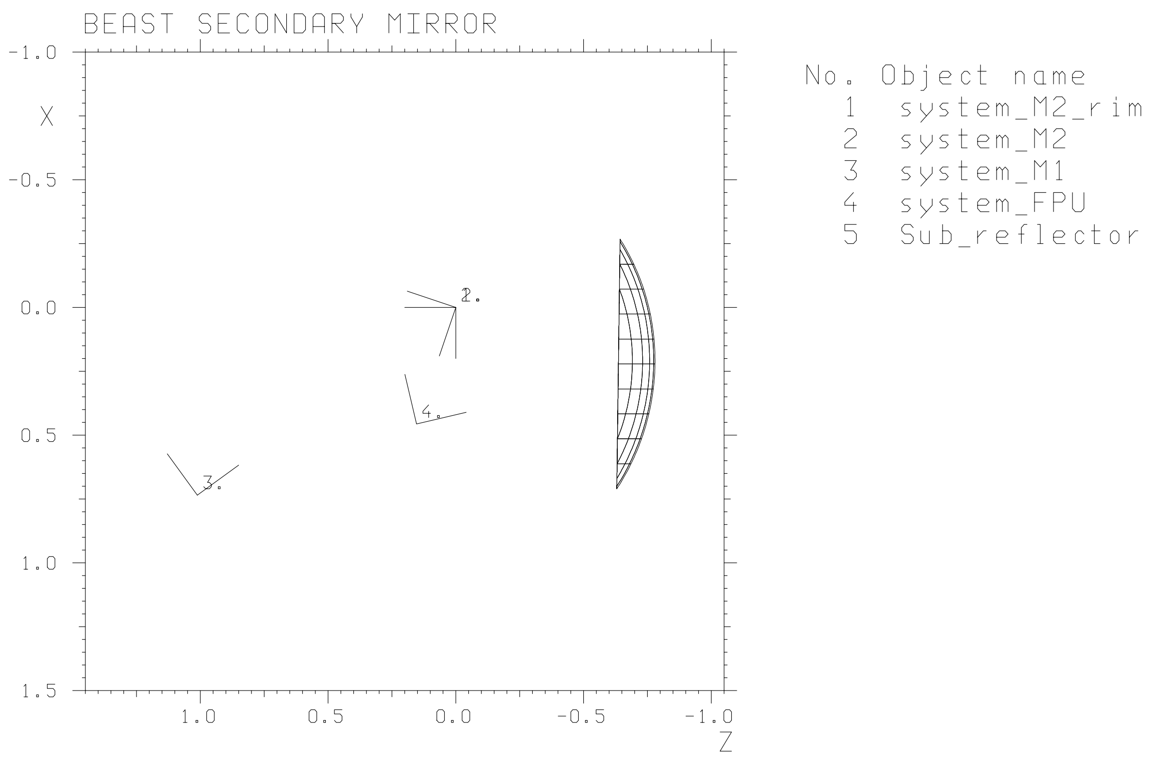

Figure 5: Secondary Mirror as seen from `system_M2_rim`

4 Focal Plane Unit

The original frame in which the focal plane has been defined is the $(X_{\text{FPU\_def}}, Y_{\text{FPU\_def}}, Z_{\text{FPU\_def}})$ with the $(Y_{\text{FPU\_def}}, Z_{\text{FPU\_def}})$ plane lies in the symmetry plane of the telescope, the $X_{\text{FPU\_def}}$ axis is pointing to the left and $Y_{\text{FPU\_def}}$ axis pointing up when the FPU is seen from the secondary mirror. As a consequence, the $Z_{\text{FPU\_def}}$ axis points at the opposite side of the secondary mirror. In this coordinate frame the phase center and the orientation of each feed is defined as reported in table 4

Horn	GHz	Phase Centre			Tilting	
		$X_{\text{FPU\_def}}$ (mm)	$Y_{\text{FPU\_def}}$ (mm)	$Z_{\text{FPU\_def}}$ (mm)	α (deg)	β (deg)
O	41.5	0.00	0.00	0.00	0.000	0.000
A	41.5	50.68	-3.54	-1.8	-0.564	-6.922
B	41.5	-50.68	-3.54	-1.8	-0.564	6.922
C	41.5	37.15	34.65	-1.5	4.592	-5.017
D	41.5	-37.15	34.65	-1.5	4.592	5.017
E	41.5	0.00	50.80	-1.4	6.704	0.000
F	30.0	26.35	-43.43	-2.1	-6.126	-3.697
G	30.0	-26.35	-43.43	-2.1	-6.126	3.697

Table 1: Position of the phase center and tilting of each feedhorn.

A new coordinate system has been defined in GRASP8:

$$\begin{aligned} X_{\text{FPU}} &= Y_{\text{FPU\_def}} \\ Y_{\text{FPU}} &= X_{\text{FPU\_def}} \\ Z_{\text{FPU}} &= -Z_{\text{FPU\_def}} \end{aligned} \tag{8}$$

so that Z_{FPU} points towards the secondary reflector, and X_{FPU} lies in the symmetry plane o the telescope. The actual orientation of the feeds in (Θ, Φ, Ψ) as defined in GRASP8 has been found in the following way. Let consider, for example the channel B. The `system_FPU` is the coordinate frame in which, at the end, the focal plane will be defined, $(X_{\text{FPU}}, Y_{\text{FPU}}, Z_{\text{FPU}})$. The `system_FPU_def` is the coordinate frame $(X_{\text{FPU\_def}}, Y_{\text{FPU\_def}}, Z_{\text{FPU\_def}})$ in which the FPU has been originally defined.

```
system_FPU                       coor_sys
(
  origin   : struct(x: 0.000000000 mm  ,y: 0.000000000 mm  ,z: 481.2900000 mm    ),
  x_axis   : struct(x: 0.5263623302    ,y: 0.000000000     ,z: -0.8502603703     ),
  y_axis   : struct(x: 0.000000000     ,y: -1.000000000    ,z: 0.000000000       ),
  base     : ref(system_M2)
)

system_FPU_def                   coor_sys
(
  origin   : struct(x: 0.000000000 mm  ,y: 0.000000000 mm  ,z: 0.000000000 mm    ),
  x_axis   : struct(x: 0.000000000     ,y: 1.000000000     ,z: 0.000000000       ),
  y_axis   : struct(x: 1.000000000     ,y: 0.000000000     ,z: 0.000000000       ),
```

```
  base    : ref(system_FPU)
)
```

The `system_FHB_1` sets the position of the phase center and the tilt around the X-axis. In this case the tilt has been set at $\Theta = -0.564°$, $\Phi = 90°$ and $\Psi = 0$ or alternatively (to get Θ always positive) $\Theta = 0.564°$, $\Phi = -90°$ and $\Psi = 0$.

```
system_FHB_1                      coor_sys
(
  origin  : struct(x: -50.68000000 mm  ,y: -3.540000000 mm  ,z: -1.800000000 mm ),
  x_axis  : struct(x: 1.000000000      ,y: -0.2966510934E-20,z: 0.6027205104E-18),
  y_axis  : struct(x: -0.2966510934E-20,y: 0.9999515516     ,z: 0.9843498011E-02),
  base    : ref(system_FPU_def)
)
```

The `system_FHB_2` sets tilt around the Y-axis. In this case the tilt has been set at $\Theta = -6.922°$, $\Phi = 0°$ and $\Psi = 0$ or alternatively $\Theta = 6.922°$, $\Phi = 180°$ and $\Psi = 0$.

```
system_FHB_2                      coor_sys
(
  origin  : struct(x: 0.000000000 mm   ,y: 0.000000000 mm    ,z: 0.000000000 mm ),
  x_axis  : struct(x: 0.9927111400     ,y: 0.000000000       ,z: 0.1205180200   ),
  base    : ref(system_FHB_1)
)
```

Finally, the X and Y axis of the `system_FHB_2` have been exchanged and the `system_FHB` has been re–defined in the `system_FPU` frame.

```
system_FHB                        coor_sys
(
  origin  : struct(x: 0.000000000 mm   ,y: 0.000000000 mm,z: 0.000000000 mm     ),
  x_axis  : struct(x: 0.000000000      ,y:    1.000000000    ,z: 0.000000000    ),
  y_axis  : struct(x: 1.000000000      ,y:    0.000000000    ,z: 0.000000000    ),
  base    : ref(system_FHB_2)
)

system_FHB                        coor_sys
(
  origin  : struct(x: -3.540000000 mm  ,y: -50.68000000 mm  ,z: 1.800000000 mm    ),
  x_axis  : struct(x: 0.9999515516     ,y: -0.2966514656E-20,z: -0.9843498011E-02),
  y_axis  : struct(x: -0.1186318890E-02,y: 0.9927111397     ,z: -0.1205121811    ),
  base    : ref(system_FPU)
)
```

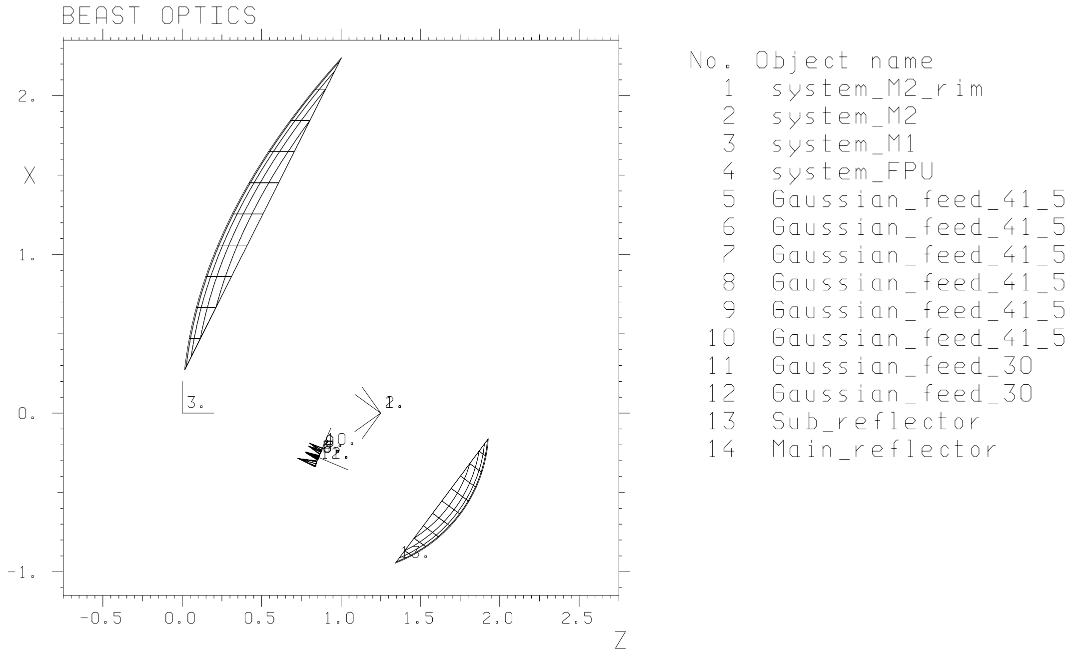

Figure 6: The BEAST Optics in GRASP8

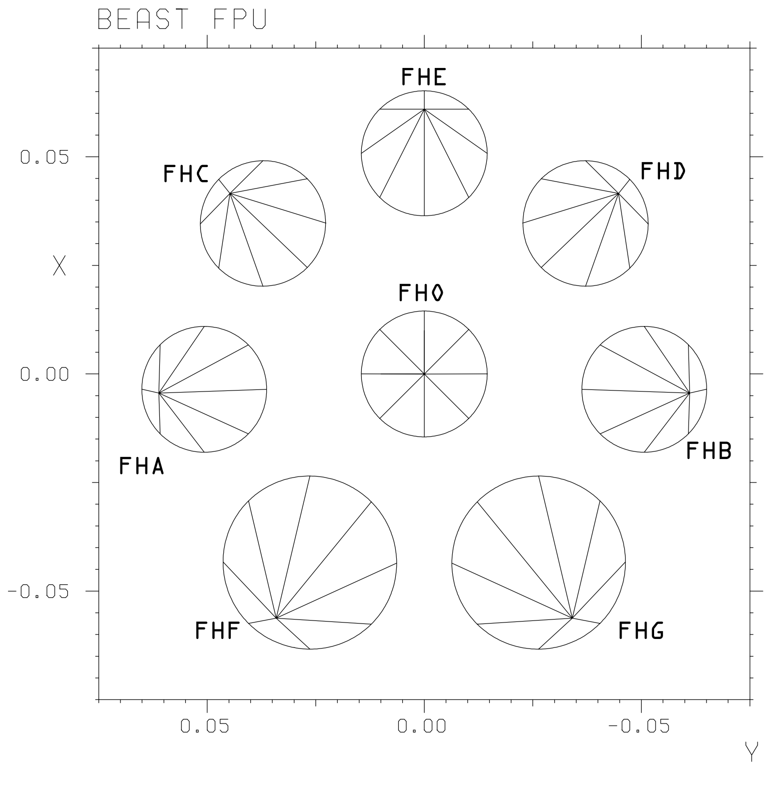

Figure 7: The BEAST Focal Plane Unit as seen from the secondary mirror. The size ho each cone depends on frequency, being the bigger at 30GHz.

5 Summary of Coordinate Systems

in table 5, the coordinate system trasformation is reported for semplicity:

System	Base	Origin			Tilting		
		X	Y	Z	Θ	ϕ	ψ
_M1		0	0	0	0	0	0
_Focal_point	_M1	0	0	1250	180	0	180
_M2	_Focal_point	0	0	0	35.41	180	0
_M2_rim	_M2	0	0	0	71.3902	0	180
_FPU	_M2	0	0	481.29	121.76	-180	-180
_FPU_def	_FPU	0	0	0	180	0	90
_FHA	_FPU	-3.54	50.68	1.8	6.9448	-85.3645	-0.0341
_FHB	_FPU	-3.54	-50.68	1.8	6.9448	85.3645	0.0341
_FHC	_FPU	34.65	37.15	1.5	6.7973	-132.3639	0.2013
_FHD	_FPU	34.65	-37.15	1.5	6.7973	132.3639	-0.2013
_FHE	_FPU	50.8	0.0	1.4	6.7040	-180.0	0.0
_FHF	_FPU	-43.43	26.35	2.1	7.1515	-31.1943	-0.1979
_FHG	_FPU	-43.43	-26.35	2.1	7.1515	31.1943	0.1979
_FHO	_FPU	0	0	0	0	0	0

Table 2: Overview of the definition of the coordinates systems. The word "system" has been intentionally omitted for semplicity.

References

[1] N. Figueirdo. Private communication, 2001.

[2] TICRA Engineering Consultant. *GRASP8 User Manual*.