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ON THE DIPOLE STRAYLIGHT CONTAMINATION IN PLANCK-LIKE CMB ANISOTROPY MISSIONS: AN ANALYTICAL APPROACH

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SUMMARY - We build a simple analytical model in order to parametrize the straylight contamination due to the kinematic CMB dipole pattern. We compute time ordered data and map from this effect. The map is analyzed in spherical harmonic expansion with a particular care to low multipoles. The impact on the observed dipole and quadrupole in discussed.

1 Introduction

It is known that the CMB kinematic dipole signal entering the main beam is subtracted away but what happens to the same signal that comes from the straylight (see, e.g. [1, 2, 3], for a discussion on straylight contamination in the context of PLANCK Low Frequency Instrument [4]) is unclear.

The aim of this note is to study this problem and to see if there are systematics on the CMB power spectrum due to this non proper subtraction of the straylight contamination from the dipole. We build a model sufficiently simple to be treated analytically [1] that allows to understand to first order how the main straylight features affect the recoverd CMB angular power spectrum in a way largely independent of the specific optical and scanning strategy detailes.

We will work in the fixed frame with the satellite with axis pointing fixed (far away) stars. In this frame the vector associated to the dipole is constant while the straylight is not (it rotates of 2π in 1 year). We consider the dipole for the motion of the Sun with respect to the rest frame of the CMB and we neglect, for simplicity, small deviations due to the motion of the Earth around the Sun.

The report is organized as follows: in Section 2 the convolution of the dipole and the straylight beam is computed and the analytical model for the beam response in the main spillover region is presented; in Section 3 the map is computed with particular care to the multipole $l = 2$; in Section 4 the computation of the power spectrum (i.e. C_l) is presented and the $l = 2$ case (i.e. the quadrupole) is discussed. Numerical estimates are presented in Section 5. Finally, our main conclusions are drawn in section 7.

[1] But not so simplistic to make the effect vanishing!

2 The dipole and the straylight beam

We start considering the convolution I of the dipole with the spillover [2]:

$$I = \int d\Omega\, T_{1m} Y_1^m(\theta,\varphi) B_{SL}(\theta,\varphi)\,, \tag{2.1}$$

where $d\Omega$ is the element of solid angle, $d\Omega = d\theta \sin\theta\, d\varphi$ with the colatitude $\theta \in [0,\pi]$ and the longitude $\varphi \in [0, 2\pi\, [$, the sum on m over $-1, 0, 1$ is understood, T_{1m} are the coefficients of the expansion of the dipole [3] on the spherical harmonics basis $Y_1^m(\theta,\varphi)$, and $B_{SL}(\theta,\varphi)$ is the beam response representing the shape of the main spillover in the (θ,φ)-plane. In this notation B_{SL} is normalized to the whole beam integrated response, dominated by the contribution in the main beam $\int_{4\pi} d\Omega B \simeq \int_{\text{main beam}} d\Omega B \simeq 2\pi\sigma_b^2$ where $\sigma_b = \text{FWHM}/\sqrt{8\ln 2}$.

The convolution I can be rewritten in the following way:

$$I = \sqrt{\frac{3}{4\pi}}\left[T_{10}\int d\theta\, d\varphi\, \sin\theta\, \cos\theta\, B_{SL}(\theta,\varphi) - \sqrt{2}\int d\theta\, d\varphi\, \sin^2\theta\, Re\left[T_{11}e^{i\varphi}\right] B_{SL}(\theta,\varphi)\right] \tag{2.2}$$

where $Re\,[...]$ stands for real part. In order to obtain the expression of eq. (2.2) it has been used that $T_{1-1} = -T_{11}^\star$ where the symbol * means complex conjugation. Moreover the following spherical harmonics (for $l = 1$) have been used [5]:

$$Y_1^0(\theta,\varphi) = \sqrt{\frac{3}{4\pi}}\cos\theta\,, \tag{2.3}$$

$$Y_1^1(\theta,\varphi) = -\sqrt{\frac{3}{8\pi}}e^{i\varphi}\sin\theta\,, \tag{2.4}$$

$$Y_1^{-1}(\theta,\varphi) = \sqrt{\frac{3}{8\pi}}e^{-i\varphi}\sin\theta\,. \tag{2.5}$$

2.1 A simple analytical model

Eq. (2.2) is general and exact (i.e. no approximation has been performed). Any specific approximation of the window function B_{SL} will introduce a certain degree of uncertainty. As already mentioned, our aim is to choose B_{SL} as simple as possible such that all the integrations are computable analytically but nevertheless without neglecting the main features that, we think, are responsible for possible systematic effects. In other words we want to simplify as much as possible this function, without vanishing the effect we are looking for.

Our approximation (of "order zero") for B_{SL} is the following:

$$\begin{aligned} B_{SL}(\theta,\varphi) = f_{SL}\,\Delta(\,\theta,\, \pi/2-\Delta_\theta,\, \pi/2+\Delta_\theta)\,\Delta(\,\varphi,\, \varphi_s-\Delta_\varphi,\, \varphi_s+\Delta_\theta) \\ \text{with}\ \ \Delta(a,b,c) = S(a-b) - S(a-c)\,, \end{aligned} \tag{2.1}$$

where f_{SL} is a constant (that is related to the ratio between the power entering the spillover and the power entering the main beam, i.e. it is a number much less than 1; in Section 5 it will be estimated) and $S(x)$ is the step function (or Heavyside function) that takes the value 1 for $x \geq 0$ and the value 0 otherwise. Eq. (2.1) is just a box, in the (θ,φ)-plane, centered around the point $(\pi/2, \varphi_s)$, with $\varphi_s = \omega_s t = 2\pi t/T$ where T is the period associated to 1 revolution around the sun (i.e. 1 yr), and with sides of length $2\Delta_\theta$ and $2\Delta_\varphi$. Notice that the point $(\pi/2, \varphi_s)$ is nothing but the direction of pointing of the straylight (or spin axis).

[2] It is the main effect of the Straylight beam in the full antenna pattern.

[3] We use the symbol T_{lm} because we want to make it clear that the dimensionality is given by a temperature ($^\circ K$). We keep a_{lm} for $\delta T/T$ i.e. a dimensionless quantity.

Note that we have assumed that the main spillover points always on the Ecliptic plane away from the Sun, a quite good approximation for PLANCK beams at least for simple scanning strategies (i.e. with the spin axis always on the Ecliptic plane away from the Sun, i.e. without spin axis precession or sinusoidal modulations).

The analytical effect of this choice is that

$$\int d\Omega\, B_{SL} = f_{SL} \int_{\frac{\pi}{2}-\Delta_\theta}^{\frac{\pi}{2}+\Delta_\theta} d\theta \sin\theta \int_{\varphi_s-\Delta_\varphi}^{\varphi_s+\Delta_\varphi} d\varphi\,, \tag{2.2}$$

and considering that

$$\int_{\frac{\pi}{2}-\Delta_\theta}^{\frac{\pi}{2}+\Delta_\theta} d\theta\, \sin\theta\, \cos\theta = 0\,, \tag{2.3}$$

$$\int_{\frac{\pi}{2}-\Delta_\theta}^{\frac{\pi}{2}+\Delta_\theta} d\theta\, \sin^2\theta = \Delta_\theta + \cos\Delta_\theta\, \sin\Delta_\theta\,, \tag{2.4}$$

$$\int_{\varphi_s-\Delta_\varphi}^{\varphi_s+\Delta_\varphi} d\varphi\, \cos\varphi = 2\cos\varphi_s\, \sin\Delta_\varphi\,, \tag{2.5}$$

$$\int_{\varphi_s-\Delta_\varphi}^{\varphi_s+\Delta_\varphi} d\varphi\, \sin\varphi = 2\sin\varphi_s\, \sin\Delta_\varphi\,, \tag{2.6}$$

we can obtain the final expression for the convolution:

$$I = -\sqrt{\frac{6}{\pi}} f_{SL} \left[\Delta_\theta + \cos\Delta_\theta \sin\Delta_\theta\right] \sin\Delta_\varphi Re\left[T_{11} e^{i\varphi_s(t)}\right]\,, \tag{2.7}$$

where we have put explicitly the dependence on the time for φ_s. Let analyze this formula:

- the constant $-\sqrt{6/\pi}$ is due to the normalization of the spherical harmonics;
- the f_{SL}-factor is due to the fact that the power entering the spillover is a "small" fraction of the power entering the main beam;
- $[\Delta_\theta + \cos\Delta_\theta \sin\Delta_\theta] \sin\Delta_\varphi$ is a volume factor due to the geometry of the B_{SL} in the (θ, φ)-plane [4];
- the last factor is depending on the angles (only φ_s) and on the coefficients of the dipole; notice that T_{10} has dropped out in this simple scheme because of the symmetry of the "box" around $\theta_s = \pi/2$.

2.2 Averages of the convolution

Integrating eq.(2.7) from 0 to T where T corresponds to 1 year we obtain

$$I_T = \int_0^T dt I(t) = 0\,. \tag{2.8}$$

This means that the convolution averages to 0 in 1 year. But the PLANCK satellite covers the full sky in $\sim$ half a year (i.e. $T/2$) then eq. (2.8) does *not* mean that there is no systematic effect on the data.

[4]Notice that in the limit of small $\Delta = \Delta_\theta = \Delta_\varphi$ this factor is $2\Delta^2$ i.e. proportional to the area [in the (θ, φ)-space] of the window function B_{SL}.

Integrating eq.(2.7) from 0 to $T/2$ we obtain

$$I_T = \int_0^{T/2} dt I(t) = \frac{T}{\pi}\sqrt{\frac{6}{\pi}} f_{SL}\left[\Delta_\theta + \cos\Delta_\theta \sin\Delta_\theta\right] \sin\Delta_\varphi Im\left[T_{11}\right] \neq 0\,, \tag{2.9}$$

where $Im[...]$ stands for imaginary part. This result is showing that we should expect a contamination of the data due to the dipole entering the spillover [in spite of the simplicity of the choice of eq. (2.1)]. Of course to have shown that, in principle, there is such a spurious effect is not enough: one has to give quantitative estimation of it in order to give prediction.

3 Building the map

The total signal that the satellite is receiving, is the sum of the two contributions:

$$T(\theta,\varphi) = T_{MB}(\theta,\varphi) + I_{SL}(\varphi)\,, \tag{3.1}$$

with

$$I_{SL}(\varphi) = \begin{cases} I(\varphi + \pi/2) \text{ for } 0 < \varphi < \pi \\ \\ I(\varphi - \pi/2) \text{ for } \pi < \varphi < 2\pi \end{cases}\,, \tag{3.2}$$

where T_{MB} is the signal entering the main beam where the dipole has been subtracted away, whereas I_{SL} is the signal due to the dipole entering the spillover. The shift in the definition of I_{SL} comes from the fact that when the main beam rotates from North to South the spin axis is shifted of $-\pi/2$ while when the main beam rotates from south to north the spin axis is shifted of $+\pi/2$. Notice that now (θ,φ) are referred to the main beam.

3.1 Computation of the T_{lm}^{SL} due to I_{SL}

As usual we expand the signal in spherical harmonics:

$$T(\theta,\varphi) = \sum_{lm} T_{lm} Y_{lm}(\theta,\varphi)\,, \tag{3.3}$$

that implies

$$T_{lm} = \int d\Omega\, T(\theta,\varphi)\, Y^\star_{lm}(\theta,\varphi)\,, \tag{3.4}$$

because of the completeness relation

$$\sum_{lm} Y^\star_{lm}(\theta,\varphi) Y_{lm}(\theta',\varphi') = \delta(\theta-\theta')\delta(\varphi-\varphi')/\sin\theta\,. \tag{3.5}$$

We start supposing that the total signal comes from I_{SL}. In other words we set, for the time being, $T_{MB} = 0$ and use eq. (3.4):

$$T_{lm}^{SL} = \int d\Omega\, I_{SL}(\varphi)\, Y^\star_{lm}(\theta,\varphi) = \int_0^{2\pi} d\varphi\, \mathcal{I}_{lm}(\varphi) I_{SL}(\varphi)\,, \tag{3.6}$$

where

$$\mathcal{I}_{lm}(\varphi) = \int_{-1}^{1} d(\cos\theta) Y^\star_{lm}(\theta,\varphi)\,. \tag{3.7}$$

3.2 The computation of T^{SL}_{1m}

Specifying $l = 1$ and $m = 0$ we obtain

$$\mathcal{I}_{10} = 0\,, \tag{3.8}$$

that of course implies $T^{SL}_{10} = 0$.

For $l = 1$ and $m = \pm 1$ we have

$$\begin{aligned} T^{SL}_{1\pm1} &= \int_0^\pi d\theta\, \sin\theta \left[\int_0^\pi d\varphi\, I(\varphi+\pi/2) Y^\star_{1\pm1}(\theta,\varphi) + \int_\pi^{2\pi} d\varphi\, I(\varphi-\pi/2) Y^\star_{1\pm1}(\theta,\varphi)\right] \\ &= \mp\sqrt{\frac{3}{8\pi}}\int_0^\pi d\theta\, \sin^2\theta \left[\int_0^\pi d\varphi\, I(\varphi+\pi/2)e^{\mp i\varphi} + \int_\pi^{2\pi} d\varphi\, I(\varphi-\pi/2)e^{\mp i\varphi}\right]\,, \end{aligned} \tag{3.9}$$

but redefining $\phi = \varphi - \pi$ we have

$$\begin{aligned} \int_\pi^{2\pi} d\varphi\, I(\varphi-\pi/2)e^{\mp i\varphi} &= \int_0^\pi d\phi\, I(\phi+\pi/2)e^{\mp i(\phi+\pi)} \\ &= -\int_0^\pi d\phi\, I(\phi+\pi/2)e^{\mp i\phi} \end{aligned} \tag{3.10}$$

then what is inside the square brackets in eq. (3.9) vanishes. We have shown that the map we are considering has no dipole contribution.

3.3 The computation of T^{SL}_{2m}

Specifying $l = 2$, it is easy to obtain

$$\mathcal{I}_{20} = \mathcal{I}_{2\pm1} = 0\,, \tag{3.11}$$

implying that also the corresponding T^{SL}_{20}, $T^{SL}_{2\pm1}$ are vanishing while

$$T^{SL}_{2\pm2} = -f_{SL}\frac{4}{3\pi}\sqrt{5}\left[\Delta_\theta + \cos\Delta_\theta \sin\Delta_\theta\right]\sin\Delta_\varphi \left(Re\left[T_{11}\right] \pm 2iIm\left[T_{11}\right]\right)\,, \tag{3.12}$$

are different from zero. These results have been computed using the definition for of spherical harmonics for $l = 2$, that we give for completeness [5]

$$Y_2^0(\theta,\varphi) = \sqrt{\frac{5}{16\pi}}(3\cos^2\theta - 1)\,, \tag{3.13}$$

$$Y_2^{\pm1}(\theta,\varphi) = \mp\sqrt{\frac{15}{8\pi}}e^{\pm i\varphi}\sin\theta\cos\theta\,, \tag{3.14}$$

$$Y_2^{\pm2}(\theta,\varphi) = \sqrt{\frac{15}{32\pi}}e^{\pm i\varphi}\sin^2\theta\,. \tag{3.15}$$

Eq. (3.12) is the (non vanishing) contribution to the quadrupole due to the dipole entering the straylight. This is one of the main result of this note.

In order to make lighter the notation we define

$$F_{SL} = f_{SL}\frac{4}{3\pi}\sqrt{5}\left[\Delta_\theta + \cos\Delta_\theta\sin\Delta_\theta\right]\sin\Delta_\varphi\,, \tag{3.16}$$

such that

$$T^{SL}_{2\pm2} = -F_{SL}\left(Re\left[T_{11}\right] \pm 2iIm\left[T_{11}\right]\right)\,. \tag{3.17}$$

4 Effects on the CMB power spectrum

From eqs. (3.1,3.4) it is clear that:

$$T_{lm} = T_{lm}^{SKY} + T_{lm}^{SL}\,, \tag{4.1}$$

where T_{lm}^{SL} is already defined in eq. (3.6) and T_{lm}^{SKY}

$$T_{lm}^{SKY} = \int d\Omega\, T_{MB}(\theta,\varphi)\, Y_{lm}^{\star}(\theta,\varphi)\,. \tag{4.2}$$

The CMB power spectrum is given by

$$C_l = \frac{1}{2l+1}\sum_{m=-l}^{l} T_{lm}^{\star} T_{lm}\,, \tag{4.3}$$

and replacing eq. (4.1) one obtains

$$\begin{aligned} C_l &= \frac{1}{2l+1}\sum_{m=-l}^{l}\left[T_{lm}^{SKY}+T_{lm}^{SL}\right]^{\star}\left[T_{lm}^{SKY}+T_{lm}^{SL}\right] \\ &= \frac{1}{2l+1}\sum_{m=-l}^{l}\left[\left(T_{lm}^{SKY}\right)^{\star}T_{lm}^{SKY}+\left(T_{lm}^{SL}\right)^{\star}T_{lm}^{SL}+\left(T_{lm}^{SL}\right)^{\star}T_{lm}^{SKY}+\left(T_{lm}^{SKY}\right)^{\star}T_{lm}^{SL}\right] \\ &\equiv C_l^{SKY}+C_l^{SL}+C_l^{SKY-SL}\,. \end{aligned} \tag{4.4}$$

4.1 The Quadrupole

In this subsection we compute eq. (4.4) for $l = 2$. We find

$$C_2^{SL} = \frac{2}{5}F_{SL}^2\left(\left[Re\left[T_{11}\right]\right]^2 + 4\,\left[Im\left[T_{11}\right]\right]^2\right)\,, \tag{4.5}$$

and

$$C_2^{SKY-SL} = -\frac{4}{5}F_{SL}\left(Re\left[T_{11}\right]Re\left[T_{22}^{SKY}\right] + 2Im\left[T_{11}\right]Im\left[T_{22}^{SKY}\right]\right)\,, \tag{4.6}$$

where it has been used that

$$Re\left[T_{2\pm2}\right] = Re\left[T_{2\mp2}\right] \tag{4.7}$$

and

$$Im\left[T_{2\pm2}\right] = -Im\left[T_{2\mp2}\right]\,. \tag{4.8}$$

5 Numerical estimates

We want to compute, in this section, what is the order of magnitude of the contribution to the quadrupole of the dipole entering the main spillover. Following eq. (2.1)

$$\int d\Omega B_{SL} = f_{SL}\,4\,\Delta_{\varphi}\sin\Delta_{\theta}\,, \tag{5.9}$$

we can obtain the parameter f_{SL}

$$f_{SL} = \frac{\int d\Omega B_{SL}}{4\,\Delta_{\varphi}\sin\Delta_{\theta}} = \frac{p}{4\,\Delta_{\varphi}\sin\Delta_{\theta}}\,, \tag{5.10}$$

where p is the relative power entering the main spillover with respect to the total (i.e. essentially entering the main beam). Choosing $\Delta_\varphi = \Delta_\theta = \pi/10$ and considering $p = 1/100$ we have $f_{SL}(p = 1/100) \simeq 2.58 \times 10^{-2}$ and $F_{SL}(p = 1/100) \simeq 4.59 \times 10^{-3}$ [see eq. (3.16)].

Moreover it is possible to show that

$$Im\,[T_{11}] = \sin\varphi_d \sin\theta_d \sqrt{\frac{2\pi}{3}}\,T\,, \tag{5.11}$$

$$T_{10} = \cos\theta_d \sqrt{\frac{4\pi}{3}}\,T\,, \tag{5.12}$$

$$Re\,[T_{11}] = -\cos\varphi_d \sin\theta_d \sqrt{\frac{2\pi}{3}}\,T\,, \tag{5.13}$$

where (θ_d, φ_d) is the direction and T is the amplitude of the dipole [5].

Considering that in our frame (Ecliptic coordinates) $(\theta_d, \varphi_d) = (1.7651\,, 2.9941)^{rad}$ and that $T = 3.346\,mK$ [6], we obtain

$$Im\,[T_{11}] = -0.21123\,mK\,, \tag{5.14}$$

$$T_{10} = -4.19925\,mK\,, \tag{5.15}$$

$$Re\,[T_{11}] = -4.73815\,mK\,, \tag{5.16}$$

We are ready to estimate the order of magnitude of C_2^{SL} and C_2^{SKY-SL}. Since T_{2m}^{SKY} are stochastic numbers with vanishing mean and standard deviation equal to C_2^{SKY}, we adopt the following relation

$$C_2^{SKY} \simeq 2\,Re\left[T_{22}^{SKY}\right]^2 = 2\,Im\left[T_{22}^{SKY}\right]^2 \tag{5.17}$$

for numerical estimate, where the factor 2 is due to the assumption that the real and imaginary part give the same contribution. We obtain:

$$C_2^{SL} = 187.5\,\mu K^2 \tag{5.18}$$

$$C_2^{SKY-SL} = \pm(385.9 \pm 32.8)\,\mu K^2 \tag{5.19}$$

where it has been chosen $C_2^{SKY} \sim 10^3 \mu K^2$ and $[Re\,[T_{11}]]^2 + 4\,[Im\,[T_{11}]]^2 \sim 4.7^2 + 4 \times 0.2^2\,mK^2$. The $\pm$ in eq. (5.19) is due to our ignorance about the relative sign of $Re\left[T_{22}^{SKY}\right]$ and $Im\left[T_{22}^{SKY}\right]$.

In Table 1 there are C_2^{SL} and C_2^{SKY-SL} for $p = 1/500\,, 1/100, 5/(100)$ and $C_2^{SKY} =$ 500, 1000, 1500 μK^2. All the numbers for C_2^{SKY-SL} contribution have to be understood with a $\pm$ in front of them (since we do not know the total sign of this contribution).

Let analyze the Table 1.

- we notice that the C_2^{SL} contribution is smaller than the C_2^{SKY-SL} contribution if p is sufficiently small. This is clear because C_2^{SL} is quadratic in p while C_2^{SKY-SL} is linear. So for $p = 1/500$ and $p = 1/100$ we have that the leading term is given by C_2^{SKY-SL} while for $p = 5/100$ we find that C_2^{SKY-SL} is subleading;
- since C_2^{SL} contribution is always positive, because of the previous observation, it is clear that there is some hope to make lower the $C_2^{SKY-SKY}$ only for the first and second column of Table 1.

[5] These relations are obtained solving the following set of equations $T = T_{1m} Y_1^m(\theta_d, \varphi_d)$, $0 = T_{1m} Y_1^m(\theta_d + \pi/2, \varphi_d)$ and $0 = T_{1m} Y_1^m(\pi/2, \varphi_d + \pi/2)$. Of course the solution can be verified replacing in $T = T(\theta_d, \varphi_d) = T_{1m} Y_1^m(\theta_d, \varphi_d)$.

$C_2(SKY)$	$p=1/500$	$p=1/100$	$p=5/100$	C_2
500	7.5	187.5	4687.6	SL
500	54.6 ± 4.6	272.8 ± 23.2	1364.4 ± 116.1	SKY-SL
1000	7.5	187.5	4687.6	SL
1000	77.2 ± 6.6	385.9 ± 32.8	1929.5 ± 164.2	SKY-SL
1500	7.5	187.5	4687.6	SL
1500	94.5 ± 8.0	472.6 ± 40.2	2363.2 ± 201.1	SKY-SL

Table 1: All the numbers are given in μK^2. See also the text.

6 Lowering of the Quadrupole

From eq. (4.4) we can rewrite the observed quadrupole C_2 as a function of F_{SL}

$$C_2(F_{SL}) = C_2^{SKY} - \frac{4}{5} B\, F_{SL} + \frac{2}{5} A\, F_{SL}^2 \,, \tag{6.20}$$

where

$$\begin{aligned} B &\equiv Re\,[T_{11}]\, Re\left[T_{22}^{SKY}\right] + 2Im\,[T_{11}]\, Im\left[T_{22}^{SKY}\right] & (6.21)\\ A &\equiv [Re\,[T_{11}]]^2 + 4\,[Im\,[T_{11}]]^2 \,. & (6.22)\end{aligned}$$

Eq. (6.20) is just a parabolic behaviour in F_{SL} with concavity in the upward direction since the coefficient of F_{SL}^2 is always positive. The sign of B is not a priori known: from eq. (4.6) or eq. (6.20) is clear that we can obtain a lowering of C_2^{SKY} only if $B > 0$. We will focus on this case [6] (that obviously implies $B/A > 0$). The minimum value $C_2(F_{SL}|_v)$ is reached by the vertex $F_{SL}|_v$,

$$F_{SL}|_v = \frac{B}{A} \,, \tag{6.23}$$

$$\begin{aligned} C_2(F_{SL}|_v) &= C_2^{SKY} - \frac{2}{5}\frac{B^2}{A} \\ &= C_2^{SKY} - \frac{2}{5}\frac{\left(Re\,[T_{11}]\, Re\left[T_{22}^{SKY}\right] + 2Im\,[T_{11}]\, Im\left[T_{22}^{SKY}\right]\right)^2}{[Re\,[T_{11}]]^2 + 4\,[Im\,[T_{11}]]^2} \,, & (6.24)\end{aligned}$$

and considering eq. (5.17) we can rewrite eq. (6.24) as

$$C_2(F_{SL}|_v) = C_2^{SKY}\left[1 - \frac{1}{5}\frac{(Re\,[T_{11}] \pm 2Im\,[T_{11}])^2}{[Re\,[T_{11}]]^2 + 4\,[Im\,[T_{11}]]^2}\right] \,, \tag{6.25}$$

where the $\pm$ is due to the fact that we do not know the relative sign of $Re\left[T_{22}^{SKY}\right]$ and $Im\left[T_{22}^{SKY}\right]$. Then the ratio between the minimum value observed for the quadrupole and the theoretical prediction is given by

$$\frac{C_2^{obs}|_{min}}{C_2^{th}} \equiv \frac{C_2(F_{SL}|_v)}{C_2^{SKY}} = \frac{4}{5}\left[1 \mp \frac{Im\,[T_{11}]}{Re\,[T_{11}]} + O\left(\frac{Im\,[T_{11}]}{Re\,[T_{11}]}\right)^3\right] \,, \tag{6.26}$$

[6] To be more precise we will choose: $-(C_2^{SKY})^{1/2} = Re\left[T_{22}^{SKY}\right] = \pm Im\left[T_{22}^{SKY}\right]$. In this discussion, the sign of $Im\left[T_{22}^{SKY}\right]$ does not play a special role, being coupled with $Im\,[T_{11}] \ll Re\,[T_{11}]$.

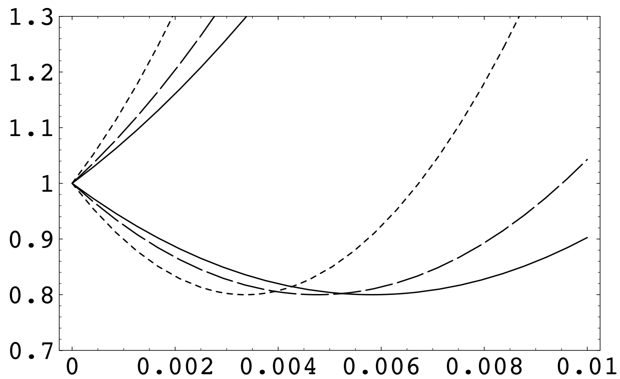

Figure 1: $y = y(F_{SL})$ with $B > 0$ (lower curves). Solid line is for $C_2^{SKY} = 1500\mu K^2$, dashed line is for $C_2^{SKY} = 1000\mu K^2$ and dotted line is for $C_2^{SKY} = 500\mu K^2$. For sake of completeness we report also the case of $B < 0$ (upper curves) which does not imply a quadrupole lowering but a quadrupole increasing.

where we have used that $Im\,[T_{11}] \ll Re\,[T_{11}]$. From eq.(6.23) and taking into account also the footnote 6, one obtains

$$p_v = 3\pi \left(\frac{C_2^{SKY}}{10}\right)^{1/2} \frac{\Delta_\varphi \sin\Delta_\theta}{\sin\Delta_\varphi\,[\Delta_\theta + \cos\Delta_\theta \sin\Delta_\theta]} \frac{|Re\,[T_{11}] \mp 2Im\,[T_{11}]\,|}{[Re\,[T_{11}]]^2 + 4\,[Im\,[T_{11}]]^2}\,, \tag{6.27}$$

and considering $\Delta_\varphi = \Delta_\theta = \Delta$ and $Im\,[T_{11}] \ll Re\,[T_{11}]$ we have the following expression

$$p_v = \frac{3\pi\Delta}{[\Delta + \cos\Delta\sin\Delta]} \left(\frac{C_2^{SKY}}{10\,Re\,[T_{11}]^2}\right)^{1/2}\,. \tag{6.28}$$

Replacing the numerical values we have

$$\frac{C_2^{obs}|_{min}}{C_2^{th}} = 0.80 \mp 0.03\,; \tag{6.29}$$

then the maximum lowering obtainable is 23 or 17 % (depending on the relative sign of the real and imaginary part of T_{22}^{SKY}). Notice that the percentage of lowering depends only on the dipole. From eq. (6.28) we have $p_v = 0.010$ with $\Delta = \pi/10$, $C_2^{SKY} = 10^3\,\mu K$. When $C_2^{SKY} = 500\,\mu K$ we obtain $p_v = 0.007$ while for $C_2^{SKY} = 1500\,\mu K$ we find $p_v = 0.012$.

In Fig. 1 we plot the following function

$$y(F_{SL}) \equiv \frac{C_2(F_{SL})}{C_2^{SKY}} \tag{6.30}$$

for $C_2^{SKY} = 0.5\times10^3\,, 10^3$ and $1.5\times10^3\,\mu K$. Since $Im\,[T_{11}] \ll Re\,[T_{11}]$ we can rewrite $y(F_{SL})$ as

$$y(F_{SL}) = 1 - \frac{4}{5}a\,F_{SL} + \frac{4}{5}a^2\,F_{SL}^2\,, \tag{6.31}$$

where

$$a = \left(\frac{Re\,[T_{11}]^2}{C_2^{SKY}}\right)^{1/2}\,. \tag{6.32}$$

As already seen in eq. (6.26), from Fig. 1 is clear that, in the simple assumption of footnote 6, the minimum of y does not depend on C_2^{SKY}. Only F_{SL} depends on C_2^{SKY}.

Setting $y(F_{SL}^\star) = 1$ we find a critical value $p^\star$

$$p^\star = \frac{3\pi}{\sqrt{5}} \frac{\sin\Delta_\varphi}{\Delta_\varphi} \frac{[\Delta_\theta + \cos\Delta_\theta \sin\Delta_\theta]}{\sin\Delta_\theta} \left(\frac{C_2^{SKY}}{Re\,[T_{11}]^2} \right)^{1/2} , \tag{6.33}$$

such that for $p < p^\star$ we obtain a lowering of the quadrupole (for $p = p^\star$ there is an exact cancelation of C_2^{SL} and C_2^{SKY-SL}, while for $p > p^\star$ there is a rising of the quadrupole).

7 Conclusion

We have built a simple analytical model aimed to parametrize the straylight contamination due to kinematic CMB dipole pattern. In the simple scheme adopted here we have shown that the map computed from this effect has no intrinsic dipole while, considering a single (or, of course, an odd number of) sky survey, it has an intrinsic quadrupole different from zero. We have demonstrated that the observed quadrupole (i.e. without correction for this effect) might be lowered by $\sim 20\%$ with respect to the intrinsic one. This aspect should be carefully considered in the context of PLANCK data analysis. We are currently studying the resulting non Gaussianity and the octupole of the pattern induced by such straylight contamination. Depending on the specific realization of the quadrupole pattern and, of course, on the level of WMAP sidelobes [7], this systematic effect might be important also for WMAP.

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