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AN ANALYTICAL APPROACH TO LOW MULTIPOLE EFFECTS FROM THE DIPOLE STRAYLIGHT CONTAMINATION IN PLANCK-LIKE CMB ANISOTROPY MISSIONS

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SUMMARY - We extend our previous analytical model aimed at the parametrization of the straylight contamination due to the kinematic CMB dipole pattern [1]. In this generalization we do not constraint the direction of pointing of the main spillover to be the same as the spin axis, but we introduce an angle $\alpha \neq 0$ between them. In this case we compute time ordered data and map. The map is analyzed in spherical harmonic expansion with a particular care to low multipoles. The impact on the dipole, quadrupole, octupole and esadecapole due to this spurious effect is discussed.

1 Introduction

In a simple analytical model [1], we tackled the systematic effect induced at low multipoles by the CMB kinematic dipole signal entering the main spillover (see, e.g. [2, 3, 4], for a discussion on straylight contamination in the context of PLANCK Low Frequency Instrument [5] or [8] in the context of WMAP).

The aim of this note is to generalize that analysis to the more realistic case where the direction of pointing of the main spillover is not parallel to from the spin axis (albeit keeping the main spillover centre in the plane defined by the telescope axis and the spin axis). We want to see how the effects at low multipoles change when there is an angle $\alpha \neq 0$ between the spin axis and the direction of the main spillover.

The model, we build here, is still fully analytical.

We will work in the rest frame with the satellite with axes pointing fixed (far away) stars. In this frame the vector associated to the dipole is constant while the straylight is not (it rotates of 2π in 1 year). We consider the dipole for the motion of the Sun with respect to the rest frame of the CMB and we neglect, for simplicity, small deviations due to the motion of the Earth around the Sun.

The report is organized as follows: in Section 2 the convolution of the dipole and the straylight beam is computed and the analytical model for the beam response in the main spillover region is presented; in Section 3 the map due to this systematic effect is analytically computed for the multipoles $l = 1, 2, 3, 4$; in Section 4 some observations on the power spectrum (i.e. C_l) are made. Finally, our main conclusions are drawn in section 5.

2 The dipole and the straylight beam

We start considering the convolution I of the dipole with the spillover [1]:

$$I = \int d\Omega \, T_{1m} Y_1^m(\theta,\varphi) B_{SL}(\theta,\varphi) \,, \tag{2.1}$$

where $d\Omega$ is the element of solid angle, $d\Omega = d\theta \sin\theta \, d\varphi$ with the colatitude $\theta \in [0,\pi]$ and the longitude $\varphi \in [0, 2\pi\,[$, the sum on m over $-1, 0, 1$ is understood, T_{1m} are the coefficients of the expansion of the dipole [2] on the spherical harmonics basis $Y_1^m(\theta,\varphi)$, and $B_{SL}(\theta,\varphi)$ is the beam response representing the shape of the main spillover in the (θ,φ)-plane. In this notation B_{SL} is normalized to the whole beam integrated response, dominated by the contribution in the main beam $\int_{4\pi} d\Omega B \simeq \int_{\rm main\ beam} d\Omega B \simeq 2\pi\sigma_b^2$ where $\sigma_b = \mathrm{FWHM}/\sqrt{8\ln 2}$.

The convolution I can be rewritten in the following way:

$$I = \sqrt{\frac{3}{4\pi}} \left[T_{10} \int d\theta \, d\varphi \, \sin\theta \, \cos\theta \, B_{SL}(\theta,\varphi) - \sqrt{2} \int d\theta \, d\varphi \, \sin^2\theta \, Re\left[T_{11} e^{i\varphi}\right] \, B_{SL}(\theta,\varphi) \right] \tag{2.2}$$

where $Re\,[...]$ stands for real part. In order to obtain the expression of eq. (2.2) it has been used that $T_{1-1} = -T_{11}^\star$ where the symbol * means complex conjugation. Moreover the following spherical harmonics (for $l = 1$) have been used [6]:

$$Y_1^0(\theta,\varphi) = \sqrt{\frac{3}{4\pi}} \cos\theta \,, \tag{2.3}$$

$$Y_1^1(\theta,\varphi) = -\sqrt{\frac{3}{8\pi}} e^{i\varphi} \sin\theta \,, \tag{2.4}$$

$$Y_1^{-1}(\theta,\varphi) = \sqrt{\frac{3}{8\pi}} e^{-i\varphi} \sin\theta \,. \tag{2.5}$$

Even if it is not clear from the notation of eq. (2.2), notice that I is a function of the geometric features of the shape of the main spillover in the (θ,φ)-plane.

2.1 A simple analytical model

Eq. (2.2) is general and exact (i.e. no approximation has yet been performed). Any specific approximation of the window function B_{SL} will introduce a certain degree of uncertainty. As already mentioned, our aim is to choose B_{SL} as simple as possible such that all the integrations are computable analytically but nevertheless without neglecting the main features that, we think, are responsible for possible systematic effects. In other words we want to simplify as much as possible this function, without vanishing the effect we are looking for.

Our approximation for B_{SL} is the following:

$$B_{SL}(\theta,\varphi) = f_{SL} \, \Delta(\,\theta, \, \theta_{ms} - \Delta_{\theta<}, \, \theta_{ms} + \Delta_{\theta>}) \, \Delta(\,\varphi, \, \varphi_{ms} - \Delta_{\varphi<}, \, \varphi_{ms} + \Delta_{\varphi>}) \quad \text{with} \quad \Delta(a,b,c) = S(a-b) - S(a-c) \,, \tag{2.6}$$

where f_{SL} is a constant (that is related to the ratio between the power entering the spillover and the power entering the main beam, i.e. it is a number much less than 1; in Section 3.7 it will be estimated) and $S(x)$ is the step function (or Heavyside function) that takes the value 1 for $x \geq 0$ and the value 0 otherwise. Eq. (2.6) is just an asymmetric rectangular box, in the

[1] It is the main effect of the Straylight beam in the full antenna pattern.

[2] We use the symbol T_{lm} because we want to make it clear that the dimensionality is given by a temperature ($^\circ K$).

(θ,φ)-plane, centered around the point $(\theta_{ms},\varphi_{ms})$ and with sides of length $\Delta_{\theta>}+\Delta_{\theta<}$ and $\Delta_{\varphi>}+\Delta_{\varphi<}$. Notice that the point $(\theta_{ms},\varphi_{ms})$ is nothing but the direction of pointing of the main spillover (that's why we put the label ms). The analytical effect of this choice is that

$$\int d\Omega\, B_{SL} = f_{SL}\int_{\theta_{ms}-\Delta_{\theta<}}^{\theta_{ms}+\Delta_{\theta>}} d\theta\sin\theta \int_{\varphi_{ms}-\Delta_{\varphi<}}^{\varphi_{ms}+\Delta_{\varphi>}} d\varphi\,. \tag{2.7}$$

Considering that

$$\int_{\theta_{ms}-\Delta_{\theta<}}^{\theta_{ms}+\Delta_{\theta>}} d\theta\,\sin\theta\,\cos\theta = \frac{1}{2}\sin\left(\delta+2\Delta_{\theta<}\right)\sin\left(\delta+2\,\theta_{ms}\right)\,, \tag{2.8}$$

$$\int_{\theta_{ms}-\Delta_{\theta<}}^{\theta_{ms}+\Delta_{\theta>}} d\theta\,\sin^2\theta = \Delta_{\theta<}+\frac{\delta}{2}-\frac{1}{2}\cos\left(\delta+2\,\theta_{ms}\right)\,\sin\left(\delta+2\,\Delta_\theta\right)\,, \tag{2.9}$$

$$\int_{\varphi_{ms}-\Delta_{\varphi<}}^{\varphi_{ms}+\Delta_{\varphi>}} d\varphi\,\cos\varphi = 2\cos\left(\varphi_{ms}+\frac{\epsilon}{2}\right)\sin\left(\Delta_{\varphi<}+\frac{\epsilon}{2}\right)\,, \tag{2.10}$$

$$\int_{\varphi_{ms}-\Delta_{\varphi<}}^{\varphi_{ms}+\Delta_{\varphi>}} d\varphi\,\sin\varphi = 2\sin\left(\varphi_{ms}+\frac{\epsilon}{2}\right)\,\sin\left(\Delta_{\varphi<}+\frac{\epsilon}{2}\right)\,, \tag{2.11}$$

with ϵ and δ implicitely defined by $\Delta_{\theta>}=\Delta_{\theta<}+\delta$ and $\Delta_{\varphi>}=\Delta_{\varphi<}+\epsilon$, we can obtain the final expression for the convolution:

$$\begin{aligned} I/f_{SL} &= T_{10}\sqrt{\frac{3}{4\pi}}\frac{1}{2}\sin\left(\delta+2\Delta_\theta\right)\sin\left(\delta+2\,\theta_{ms}\right)\left(2\Delta_\varphi+\epsilon\right) \\ &\quad -4\sqrt{\frac{3}{8\pi}}\left(\Delta_\theta+\frac{\delta}{2}-\frac{1}{2}\cos\left(\delta+2\,\theta_{ms}\right)\,\sin\left(\delta+2\,\Delta_\theta\right)\right) \\ &\quad \cdot\left[Re\left[T_{11}\right]\cos\left(\varphi_{ms}+\frac{\epsilon}{2}\right)-Im\left[T_{11}\right]\sin\left(\varphi_{ms}+\frac{\epsilon}{2}\right)\right]\sin\left(\Delta_\varphi+\frac{\epsilon}{2}\right)\,. \end{aligned} \tag{2.12}$$

Here we have made the notation ligther setting $\Delta_{\theta<}\equiv\Delta_\theta$ and $\Delta_{\varphi<}\equiv\Delta_\varphi$. If the box is symmetric (i.e. $\delta=0$ and $\epsilon=0$) and if the direction of the main spillover coincides with the spin axis (i.e. $(\theta_{ms},\varphi_{ms})=(\pi/2,\varphi_s)$), then the convolution becomes [1]

$$I=-\sqrt{\frac{6}{\pi}}f_{SL}\left[\Delta_\theta+\cos\Delta_\theta\sin\Delta_\theta\right]\sin\Delta_\varphi Re\left[T_{11}e^{i\varphi_s(t)}\right]\,, \tag{2.13}$$

where we have put explicitly the dependence on the time for φ_s. Notice that the term proportional to T_{10} has dropped out in this simple case. In general, the T_{10}-component does not appear if $\delta+2\theta_{ms}(t)=n\pi$ where n is an integer. If $\theta_{ms}(t)=\pi/2-\beta(t)$ we have a perfect vanishing (for $n=1$) when $\beta(t)=\delta/2$. This means that each time $t=\beta^{-1}(\delta)$ we have a compensation between the direction of the main spillover and the asymmetry of the box such that the coefficient of T_{10}-term, is vanishing.

2.2 Relation between main beam and main spillover

During the rotation of the main beam, the main spillover, if it is not lying on the spin axis, draws a cone (see Fig.1). This means that the direction of the main spillover and of the main beam are related by

$$\theta_{ms} = \frac{\pi}{2}-\tan^{-1}\left(\tan\alpha\,\cos\theta_{mb}\right)\,, \tag{2.14}$$

$$\varphi_{ms} = \varphi_s+\tan^{-1}\left(\tan\alpha\,\sin\theta_{mb}\right)\,, \tag{2.15}$$

where θ_{mb} is the colatitude of the main beam, α is the angle at the vertex of the cone and φ_s is the longitude of the spin axis (see also caption of Fig.1).

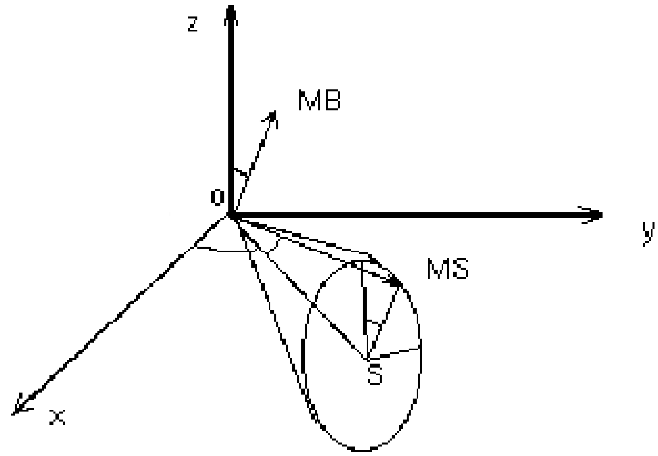

Figure 1: Cone drawn by the direction of the main spillover (MS) during the rotation of the main beam (MB). θ_{mb} is the angle between z axis and MB, φ_s is the angle in x-y plane between x axis and spin axis (OS) and α is the angle between OS and MS.

In order to make realistic this simple model, we have to check that during the rotation, the solid angle, subtended by the main spillover (Ω_{ms}), is constant. A simple computation gives

$$\Omega_{ms} = 4\Delta \sin\Delta \sin\theta_{ms} = \frac{4\Delta\sin\Delta}{\sqrt{1+\tan^2\alpha\,\cos^2\theta_{mb}}}\,, \tag{2.16}$$

where (for simplicity) the square box (for the main spillover) in the beam pattern has been considered. Of course eq. (2.16) is not constant because θ_{mb} is a function depending on time. But in the limit of small α we obtain

$$\Omega_{ms} = 4\Delta\sin\Delta\left[1-\frac{1}{2}\cos^2\theta_{mb}\alpha^2+\mathcal{O}\left(\alpha^4\right)\right]\,, \tag{2.17}$$

that is constant (i.e. does not depend on θ_{mb}) at 0th and 1st order in α. This means that the computation will be done up to linear order in α. Then eqs. (2.14,2.15) will be Taylor expanded for small α, obtaining

$$\theta_{ms} = \frac{\pi}{2} - \cos\theta_{mb}\,\alpha + \mathcal{O}\left(\alpha^3\right)\,, \tag{2.18}$$

$$\varphi_{ms} = \varphi_s + \sin\theta_{mb}\,\alpha + \mathcal{O}\left(\alpha^3\right)\,. \tag{2.19}$$

3 Building the map

The total signal that the satellite receives, is the sum of the two contributions:

$$T(\theta,\varphi) = T_{MB}(\theta,\varphi) + I_{SL}(\theta,\varphi)\,, \tag{3.1}$$

where T_{MB} is the signal entering the main beam where the dipole has been subtracted away, whereas I_{SL} is the signal due to the dipole entering the spillover. For the first survey I_{SL} is given by

$$I_{SL}^{(I)}(\theta,\varphi) = \begin{cases} I(\pi/2-\alpha\cos\theta, \varphi+\pi/2-\alpha\sin\theta) \text{ for } & 0<\varphi<\pi \\ I(\pi/2-\alpha\cos\theta, \varphi-\pi/2+\alpha\sin\theta) \text{ for } & \pi<\varphi<2\pi \end{cases}\,, \tag{3.2}$$

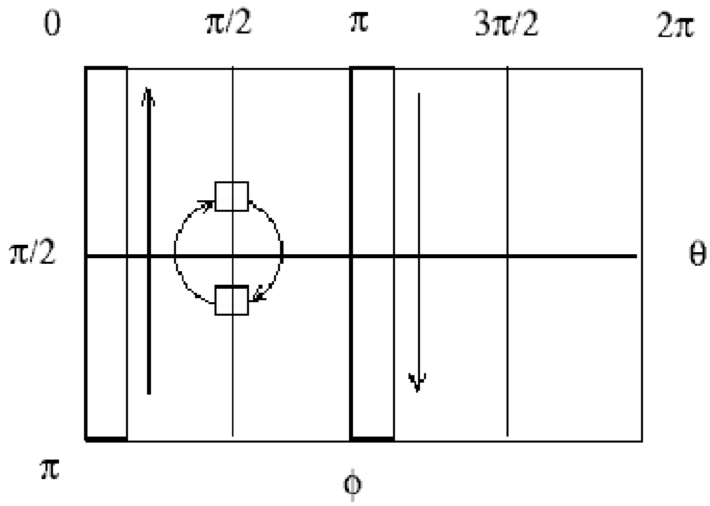

Figure 2: Sketch of the scanning during the first survey.

while, for the second survey

$$I^{(II)}_{SL}(\theta,\varphi) = \begin{cases} I(\pi/2-\alpha\cos\theta, \varphi-\pi/2+\alpha\sin\theta) \text{ for } \quad 0<\varphi<\pi \\ I(\pi/2-\alpha\cos\theta, \varphi+\pi/2-\alpha\sin\theta) \text{ for } \quad \pi<\varphi<2\pi \end{cases} . \tag{3.3}$$

The shift in the definition of I_{SL} (during either the first or the second survey) comes from the fact that when the main beam rotates from North to South the main spillover is shifted of $-\pi/2$ plus a small correction proportional to α (due to the non perfect alignment of the main spillover with the spin axis) while when the main beam rotates from South to North the main spillover is shifted of $+\pi/2$ minus a small correction proportional to α (still due to the non perfect alignment of the main spillover with the spin axis). See Fig.2 for a sketch of the scanning.

Notice that now (θ,φ) are referred to the main beam (we omitted the label mb to make the notation ligher).

3.1 Computation of the T^{SL}_{lm} due to I_{SL}

As usual we expand the signal in spherical harmonics:

$$T(\theta,\varphi) = \sum_{lm} T_{lm} Y_{lm}(\theta,\varphi)\,, \tag{3.4}$$

that implies

$$T_{lm} = \int d\Omega\, T(\theta,\varphi)\, Y^{\star}_{lm}(\theta,\varphi)\,, \tag{3.5}$$

because of the completeness relation

$$\sum_{lm} Y^{\star}_{lm}(\theta,\varphi) Y_{lm}(\theta',\varphi') = \delta(\theta-\theta')\delta(\varphi-\varphi')/\sin\theta\,, \tag{3.6}$$

or

$$\int d\Omega\, Y^{\star}_{lm}(\theta,\varphi) Y^{\star}_{l'm'}(\theta,\varphi) = \delta_{ll'}\delta_{mm'}\,. \tag{3.7}$$

We start supposing that the total signal comes from I_{SL}. In other words we set, for the time being, $T_{MB} = 0$ and use eq. (3.5):

$$T_{lm}^{SL} = \int d\Omega \, I_{SL}(\theta, \varphi) \, Y_{lm}^{\star}(\theta, \varphi) \, , \tag{3.8}$$

where we rewrite the convolution as

$$I(\theta, \varphi) = c_1 \sin(2\theta) - (c_2 - c_3 \cos(2\theta)) \, (d_1 \cos\varphi - d_2 \sin\varphi) \ , \tag{3.9}$$

with

$$\begin{aligned}
c_1 &= \sqrt{3/4\pi} \, f_{SL} \Delta \, \sin(2\Delta) \, T_{10} & (3.10)\\
c_2 &= 4\sqrt{3/8\pi} \, f_{SL} \, \Delta & (3.11)\\
c_3 &= 4\sqrt{3/8\pi} \, f_{SL} \, \sin(2\Delta)/2 & (3.12)\\
d_1 &= \sin\Delta \, Re \, [T_{11}] & (3.13)\\
d_2 &= \sin\Delta \, Im \, [T_{11}] \ . & (3.14)
\end{aligned}$$

Here it has been chosen $\epsilon = \delta = 0$ and $\Delta_{\theta>} = \Delta_{\theta<} = \Delta_{\varphi>} = \Delta_{\varphi<} = \Delta$ in eq. (2.12).

3.2 The computation of T_{00}^{SL}

Specifying $l = 0$ and $m = 0$ we obtain

$$T_{00}^{SL} = \frac{4}{\sqrt{\pi}} d_1 (c_2 + c_3) + \mathcal{O}\left(\alpha^2\right) \, . \tag{3.15}$$

The monopole does not change up to the first order in α. Thus, we recover the result of Ref. [1].

It is possbile to show that repeating the computation for the second survey [i.e. taking into account $I_{SL}^{(II)}$ given in eq. (3.3)], one obtains $T_{00}^{SL,(II)} = -T_{00}^{SL}$. This means that the average of the map (for the monopole) on two surveys is zero. This is in agreement with the results of [1] because the computation at the linear order is equal to the computation at zeroth order (i.e. $\alpha = 0$).

3.3 The computation of T_{1m}^{SL}

Specifying $l = 1$ and $m = 0$ we obtain

$$T_{10}^{SL} = \sqrt{\frac{3}{4\pi}} 2\pi c_1 4 \frac{\alpha}{3} \, . \tag{3.16}$$

For $l = 1$ and $m = \pm 1$ we have

$$T_{1\pm1}^{SL} = \sqrt{\frac{3}{8\pi}} \, (\pm d_1 + i d_2) \, (c_2 + c_3) 4\pi \frac{\alpha}{3} \, . \tag{3.17}$$

This means that only in the simplified case [in which the spin axis is parallel to the direction of the main spillover (i.e. $\alpha = 0$)] we have that the map we are considering has no dipole contribution. Since the dipole is important for the calibration, this systematic effect deserves further investigation.

In this case repeating the computation for the second survey [i.e. using eq. (3.3)] it is possible to show that one obtain the same result as the first survey: $T_{1m}^{SL,(II)} = T_{1m}^{SL}$ for every m. Thus, the average of the map (for the dipole) on the two surveys is exactly given by one single survey.

3.4 The computation of T^{SL}_{2m}

Specifying $l = 2$ and $m = 0$, it is easy to obtain

$$T^{SL}_{20} = \mathcal{O}\left(\alpha^2\right) , \tag{3.18}$$

implying that it is vanishing at linear order in α. For $m = \pm 1$:

$$T^{SL}_{2\pm 1} = 0 , \tag{3.19}$$

where no expansion in α has been performed in order to obtain this result. For $m = \pm 2$:

$$T^{SL}_{2\pm 2} = -\left(\frac{4}{3}\right)^2 \sqrt{\frac{15}{32\pi}} \left(d_1 \pm 2id_2\right)\left(c_2 + c_3\right) , \tag{3.20}$$

where α does not appear at linear order [there are corrections of order $\mathcal{O}\left(\alpha^2\right)$].

These results have been computed using the definition for of spherical harmonics for $l = 2$, that we report here for sake of completeness [6]

$$Y_2^0(\theta,\varphi) = \sqrt{\frac{5}{16\pi}}(3\cos^2\theta - 1) , \tag{3.21}$$

$$Y_2^{\pm 1}(\theta,\varphi) = \mp\sqrt{\frac{15}{8\pi}}e^{\pm i\varphi}\sin\theta\cos\theta , \tag{3.22}$$

$$Y_2^{\pm 2}(\theta,\varphi) = \sqrt{\frac{15}{32\pi}}e^{\pm 2i\varphi}\sin^2\theta . \tag{3.23}$$

Eq. (3.20) is the (non vanishing) contribution to the quadrupole due to the dipole entering the straylight. This is one of the main result of this note. We note that in the linear approximation it does not show any dependence on α recovering the result obtained in [1].

3.5 The computation of T^{SL}_{3m}

Setting $l = 3$, $m = 0$ and $m = \pm 1$, one obtains

$$T^{SL}_{30} = T^{SL}_{3\pm 1} = \mathcal{O}\left(\alpha^3\right) , \tag{3.24}$$

For $m = \pm 2$ and $m = \pm 3$.

$$T^{SL}_{3\pm 2} = T^{SL}_{3\pm 3} = 0 , \tag{3.25}$$

without expanding in α. This means that there is no contribution to the octupole.

These results have been computed using the spherical harmonics for $l = 3$, that we report here for sake of completeness [6]

$$Y_3^0(\theta,\varphi) = \frac{1}{4}\sqrt{\frac{7}{\pi}}(-3\cos\theta + 5\cos^3\theta) , \tag{3.26}$$

$$Y_3^{\pm 1}(\theta,\varphi) = \mp\frac{1}{8}\sqrt{\frac{21}{\pi}}e^{\pm i\varphi}\sin\theta\left(-1 + 5\cos^2\theta\right) , \tag{3.27}$$

$$Y_3^{\pm 2}(\theta,\varphi) = \frac{1}{4}\sqrt{\frac{105}{2\pi}}e^{\pm 2i\varphi}\sin^2\theta\cos\theta , \tag{3.28}$$

$$Y_3^{\pm 3}(\theta,\varphi) = \mp\frac{1}{8}\sqrt{\frac{35}{\pi}}e^{\pm 3i\varphi}\sin^3\theta . \tag{3.29}$$

3.6 The computation of T_{4m}^{SL}

Setting $l = 4$, $m = 0$ and $m = \pm 1$, one obtains

$$T_{40}^{SL} = T_{4\pm 1}^{SL} = 0\,, \tag{3.30}$$

For $m = \pm 2$:

$$T_{4\pm 2}^{SL} = -\frac{1}{2}\sqrt{\frac{5}{2\pi}}\,(d_1 \pm 2id_2)\,\frac{8}{15}\,(c_2 + c_3) + \mathcal{O}\left(\alpha^2\right)\,. \tag{3.31}$$

For $m = \pm 3$:

$$T_{4\pm 3}^{SL} = 0\,. \tag{3.32}$$

For $m = \pm 4$:

$$T_{4\pm 4}^{SL} = -\frac{12}{225}\sqrt{\frac{35}{2\pi}}\,(d_1 \pm 4id_2)\,(c_2 + c_3) + \mathcal{O}\left(\alpha^2\right)\,. \tag{3.33}$$

These results have been computed using the definition for of spherical harmonics for $l = 4$, that we report here for sake of completeness [6]

$$\begin{aligned}
Y_4^0(\theta,\varphi) &= \frac{3}{16\sqrt{\pi}}(3 - 30\cos^2\theta + 35\cos^4\theta)\,, && (3.34)\\
Y_4^{\pm 1}(\theta,\varphi) &= \mp\frac{3}{8}\sqrt{\frac{5}{\pi}}e^{\pm i\varphi}\cos\theta\sin\theta\left(-3 + 7\cos^2\theta\right)\,, && (3.35)\\
Y_4^{\pm 2}(\theta,\varphi) &= \frac{3}{8}\sqrt{\frac{5}{2\pi}}e^{\pm 2i\varphi}\sin^2\theta\left(-1 + 7\cos^2\theta\right)\,, && (3.36)\\
Y_4^{\pm 3}(\theta,\varphi) &= \mp\frac{3}{8}\sqrt{\frac{35}{\pi}}e^{\pm 3i\varphi}\cos\theta\sin^3\theta\,, && (3.37)\\
Y_4^{\pm 4}(\theta,\varphi) &= \frac{3}{16}\sqrt{\frac{35}{2\pi}}e^{\pm 4i\varphi}\sin^4\theta\,. && (3.38)
\end{aligned}$$

3.7 Comparison among C_l^{SL} with low l

As in [1] we consider

$$f_{SL} = \frac{\int d\Omega B_{SL}}{4\,\Delta\sin\Delta} = \frac{p}{4\,\Delta\sin\Delta}\,, \tag{3.39}$$

where p is the relative power entering the main spillover with respect to the total (i.e. essentially entering the main beam). By the definition of C_l^{SL}:

$$C_l^{SL} = \frac{1}{2l+1}\sum_{m=-l}^{l}\left(T_{lm}^{SL}\right)^\star T_{lm}^{SL}\,, \tag{3.40}$$

we compute

$$C_0^{SL} = 6\left(\frac{p}{\pi}\right)^2 f(\Delta)Re\left[T_{11}\right]^2\,, \tag{3.41}$$

$$C_1^{SL} = \frac{\alpha^2 p^2}{3}\left[\cos^2\Delta\, T_{10}^2 + \frac{1}{2}f(\Delta)|T_{11}|^2\right]\,, \tag{3.42}$$

$$C_2^{SL} = \frac{2}{9}\left(\frac{p}{\pi}\right)^2 f(\Delta)\left(Re\,[T_{11}]^2 + 4Im\,[T_{11}]^2\right)\,, \tag{3.43}$$

$$C_4^{SL} = \frac{1}{9}\left(\frac{p}{\pi}\right)^2 \frac{64}{375} f(\Delta)\left(Re\,[T_{11}]^2 + \frac{53}{8} Im\,[T_{11}]^2\right)\,, \tag{3.44}$$

where

$$f(\Delta) = \left(1 + \frac{\sin\Delta}{\Delta}\cos\Delta\right)^2\,,$$

We choose $\Delta = \pi/10$ and $p = 1/100$. Moreover it is possible to show [1, 7] that

$$Im\,[T_{11}] = 0.69823\,mK\,, \tag{3.45}$$
$$T_{10} = -1.32225\,mK\,, \tag{3.46}$$
$$Re\,[T_{11}] = 4.69963\,mK\,. \tag{3.47}$$

Here we give some numerical results. For $\alpha = \pi/36$ we have

$$C_0^{SL} = 0.0050299\,mK^2 \tag{3.48}$$
$$C_1^{SL} = 0.000011135\,mK^2 \tag{3.49}$$
$$C_2^{SL} = 0.00020274\,mK^2 \tag{3.50}$$
$$C_4^{SL} = 0.000018222\,mK^2 \tag{3.51}$$

and then the ratios:

$$C_0^{SL}/C_1^{SL} = 451.73 \tag{3.52}$$
$$C_2^{SL}/C_1^{SL} = 18.208\,. \tag{3.53}$$
$$\tag{3.54}$$

For $\alpha = \pi/18$ we have

$$C_0^{SL} = 0.0050299\,mK^2 \tag{3.55}$$
$$C_1^{SL} = 0.000044539\,mK^2 \tag{3.56}$$
$$C_2^{SL} = 0.00020274\,mK^2 \tag{3.57}$$
$$C_4^{SL} = 0.000018228\,mK^2 \tag{3.58}$$

and then the ratios:

$$C_0^{SL}/C_1^{SL} = 112.93 \tag{3.59}$$
$$C_2^{SL}/C_1^{SL} = 4.5520\,. \tag{3.60}$$
$$\tag{3.61}$$

4 Effects on the CMB power spectrum

Some qualitative considerations on the low multipoles are listed here.

- The monopole is not touched at all by the shift of the spin axis up to the first order in α,
- We note that contrarily to what happened in [1], now, with $\alpha \neq 0$, there is a contamination of the dipole itself. This could have some important consequences on calibrations based on the kinematic dipole.

- For what concerns the quadrupole, at the order we are performing the expansion (linear in α in temperatures and quadratic in α in angular power spectrum) the analysis we did in [1] is still fully valid, since the computation with $\alpha \neq 0$ is the same as $\alpha = 0$.
- The octupole is not touched at all by this effect.
- The esadecapole presents a contamination due to this systematic effect that does not depend on α (at the order we are performing the computation).

5 Conclusion

We have extended our previous analytical model [1] where the straylight contamination due to kinematic CMB dipole pattern was studied. The generalization we took into account is given by the splitting between the direction of the main spillover and the spin axis (for a simple case with the main spillover centre in the plane defined by the telescope axis and the spin axis). We called α the angle between them.

We have shown that the map computed from this effect has intrinsic dipole (proportional to α) while the intrinsic quadrupole is untouched by α at first order. Moreover we have seen that there is no intrinsic octupole (even with $\alpha \neq 0$) while there is an intrinsic esadecapole (even with $\alpha = 0$).

Since the quadrupole is untouched by this generalization, the analysis we did in [1] remains valid also in this case where spin axis and direction of the main spillover are not parallel.

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