Internal Report IASF-BO 421/2005

November 7, 2005

COSMOLOGICAL APPLICATIONS OF A NUMERICAL CODE FOR THE SOLUTION OF THE KOMPANEETS EQUATION

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SUMMARY – In this report we present some remarkable examples of the numerical solution obtained with the up-dated version of our code for the Kompaneets equation in cosmological context.

1 Introduction

Considering the results supplied by the accuracy tests, it appears clear that KYPRIX could be a very accurate, versatile, and useful numerical code to study the physical processes happened in the primordial plasma and the related energies. Anyway, this code could be used for any problem you can reconduce to the Kompaneets equation in cosmological scenario: the computation of the photons distribution function is the case on wich we will stop ourself.
Here after will be exposed the results obtained by the numerical integration of the Kompaneets equation. In particular, we will focus on the results obtained about the evolution of the CMBR spectrum in terms of brigtness temperature. Then will be presented a deep analysis of the evolution with iniutial conditions represented by a Bose-Einstein spectrum and by a superposition of blackbodies.

2 Distortions at $z \sim z_{BE}$

As well known, kinetic equilibrium between matter and radiation is achieved in a scale time given by

$$\tau_C = T_{\gamma e} \frac{m_e c^2}{k_B T_e}\,, \tag{1}$$

where $T_{\gamma e} = 1/(n_e \sigma_T c)$ is the electron-photon collision time. In all the analysis done in this work we have ignored complication to the cosmic evolution (like radiative particles decay or changes in the state equation of particles lighter than the electron).
From the analysis of primordial phases from wich we can obtain informations about the CMBR spectrum, appears that for $z > z_{term} \sim \textit{some units} \times (10^6 - 10^7)$ (the exact value depends on the barion density and H_0) a distortion caused by an energy injection of any entity would be cancelled cause of the high efficiency of the photons producer processes, whose coupled to the Compton scattering effect they can thermalize the CMBR spectrum.
After this age, in a generalized manner, the thermalization is not totally achievable. The CMBR spectrum, in this age of the cosmic evolution, is well described by a Bose-Einstein spectrum:

$$\eta_{BE} = \frac{1}{e^{x/\phi_{BE}+\mu} - 1}\,, \tag{2}$$

where $\mu(x) = \mu_0 e^{-x_c(z_{BE})/(x/\phi_{BE})}$ is the dimensioneless chemical potential, $\phi_{BE} \simeq (1 - 1.11\mu_0)^{-1/4}$ and $\mu_0 \simeq 1.4\,\Delta\epsilon/\epsilon_i$ in the limit where $\mu_0, \Delta\epsilon/\epsilon_i \ll 1$. The dimensioneless frequency x_c is the solution to the equation $t_{abs} = t_C$ computed at $z = z_{BE}$, where t_{abs} is the combined absorption scale time of Bremsstrahlung and radiative Compton (for an approximated expression of x_c see Burigana *et al.* 1991).
The contribution of radiative Compton is very efficient (much more than Bremsstrahlung) at very high redshifts (Gould, 1972; Danese & De Zotti, 1977). In order to appriciate this effect we started the integration from $z \simeq 6.6 \times 10^5$, that is to say from $y(z) \simeq 5$, as highest value for z, to procede then to minor values, in order to monitor a decreasing in the production photons rate caused by radiative Compton (fig.).
The integration interval of the two following cases starts from $z = 6.6 \times 10^5$ and ends today.

The introduction of the cosmological constant doesn't bring relevant differences in the evolution of the CMBR spectrum starting from a Bose-Einstein one. This fact is enuogh intuitive, because the cosmological constant supplies to the evolution of the universe in a substantial manner only at low redshifts. Given that a Bose-Einstein spectrum could be established only for redshift $z \gtrsim z_{BE}$ (z_{BE} indicates the end of the Bose-Einstein age), it follows that the formation and the evolution of BE-like spectrum don't show any variation with considering Λ in the evolution of the scale factor in the early universe.

3 Distortions at $z < z_{BE}$

The ages analyzed in the previous and the next sections are two limit cases: in fact, a solution to the Kompaneets equation could be obtained analytically, using some approximations (Burigana *et al*. 1995).
The age following the Bose-Einstein one is that in wich the only way to reach a solution to the Kompaneets equation, in cosmological scenarios, is the numerical one. Anyway, some analytical approximation have been obtained (Burigana, Danese, De Zotti 1995), but these are good only under determinated conditions, so they haven't a general validity (even if the treated case has some relevant importance, being representative of numerous cosmological and astrophysics context).
The initial photons spectrum (the spectrum immediately after the energy injection) is supposed to be a superposition of blackbodies:

$$\eta(x, y^*) = (4\pi y^*)^{-1/2} \int_0^\infty \eta_0(x') \exp\left[- \frac{(\ln(x/x') + 3y^*)^2}{4y^*} \right] \frac{\mathrm{d}x'}{x'}\,, \tag{3}$$

where $\eta_0 = 1/[exp(x'/\phi_i) - 1]$, $\phi_i = T_i/T_r \simeq (1 + \Delta\epsilon/\epsilon_i)^{-1/4}$, with ϵ_i being the impertubated radiation energy density and y^* is the comptonization parameter.
Anyway, in the primordial ages of the universe there are many processes able to modify the CMBR spectrum in a such way. The temperature distribution function given by the previous equation corresponds to a comptonization of a Plackian spectrum made by hot electrons (Zel'dovich & Sunyaev, 1969). However, in the case of small distortions, the shape of the distorted spectrum is widely independent from a detailed description of the temperature distribution function.
The redshift interval in wich the heating process could be happened is wide. Here are shown some results, in wich we consider two epochs where the process could be happened. Except the cosmological constant, that could be considered null in some cases, the cosmological parameters used are those supplies by the WMAP mission.
The final CMBR spectrum shows an intermediate shape between the one corresponding to a comptonized spectrum and a BE-like spectrum. The presence (and the entity) of a depression that could remember a Bose-Einstein spectrum depends strongly on the epoch when the energy injection occurred.

4 Distortions at $z_{rec} \lesssim z \ll z_{BE}$

For this case, we started the integration from a time such to have $y(z) < 1$, considering different values of injected fractional energy. The results provided by the analytic approximation, obtained by Burigana, Danese & De Zotti (1995), for this limit case, fit in a very good manner with the values of η obtained through the numerical integration of the Kompaneets equation.
The initial spectrum is assumed to be a superposition of blackbodies. In this section are shown the evolutions of superpositions of blackbodies starting from different cosmic epochs and with different input parameters. We will focus our attention on: presence (or absence)

of a cosmological constant component; different integration interval.
The differences between the spectra shapes are determined by the epoch in wich the injection accurs. At high frequencies ($\nu > 10$ cm), see fig. , the spectrum is in equilibrium: this case doesn't show any kind of evolution (the comptonization parameter is very small: $y \simeq 5 \times 10^{-3}$), while in the spectra in fig. we can see a depression at intermediate frequencies (Rayleigh-Jeans depression) and a brightness temperature reduction at high frequencies. Both the features are dued to the Compton scattering effect ($y \simeq 2 \times 10^{-2}$).

5 Distortion produced during the reionization of the universe

WMAP is one of the most important cosmological mission realized in the last years. The results obtained through it, have a great precision and this permits to put some constraint on the cosmological parameters of the standard model. In this section, we will focus our attention on the consequence related to a particular cosmological parameter: the optical depth of the universe. The value obteined from WMAP data is $\tau_{reion} = 0.17 \pm 0.04$, that implies a reionization happened at $z_{reion} = 20^{+10}_{-9}$. This phenomenon could be dued to a primordial star formation (star of popolation III, for example). Thermonuclear reactions produced by the gravitational collapse of the matter could heat the whole universe, producing ionization of the surrounding matter of the collapsed one. A simil process produces a particular effect: the last scattering surface (LSS) seems to be shifted forward in time, that is to say it was happened a new scattering process. The consequence of this on the CMBR are simply imaginable: heating of photons implies a strong redistribution of their energies, so the lost of information about the CMBR spectrum will be huge.
The anisotropies were be suppressed in particula at small angle scale: in absence of motion the elctrons don't have privileged directions and the diffusion could generate a more omogeneus and a more isotropic radiation distribution. On the other hand, a new LSS implies a generation of new anisotropies, but in order to have a good visibility of them the reionization process would be happened more back in time than the data indicates.
Since the discover of this possible reionization, it had been proposed many models in order to have the value found for τ_{reion}. A significative contribution ($\sim 0.05 - 0.07$) to this value comes from an ionization related to a photons production and to hot gas injection from primordial galaxies and quasars at relatively low redshifts ($z \lesssim z_{reion} \sim 5 - 6$), where the direct observation of the Gunn-Peterson effect could tell us something about the ionization level of the intergalactic medium at that age. The contribution left over ($\sim 0.05 - 0.12$) of the observed value of τ_{reion} could be related to precesses happened at higher redshifts.
The signs that a reionization can leave could be divided in three categories:
i) decreasing of power for anisotropies at high multipole orders because of photons diffusion;
ii) increasing of power for anisotropies in the polarization-temperature cross-correlation and in the E-mode spectrum, considering the possibility that the last could be marked at low and mid multipoles (this fact is related to the epoch in wich this "second last scattering" could happen);
iii) generation of free-free-like and comptonization-like spectral distortion, related to the heating of the intergalactic medium during the reionization.
The last effect depends strongly from the thermal history of the medium, not only from its ionization level: this is dued to the fact that energy exchange between matter and radiation depends directly from the elctronic temperature.
But let's see the results obtained through KYPRIX. We started the integration from relatively recent epochs ($z \sim 20$) and we focused out analysis on the dependence of the spectral distortions from the cosmological constant.

The initial conditions is represented by a Planckian distribution at $T = 2.725°K$, in order to simulate the process we introduce $\phi \equiv T_e/T_r = 10^4$ and the input cosmological parameters are:$h = 0.68, \Omega_b = 0.047, \Omega_m = 0.029, \Omega_\Lambda = 0.73$ for the case corresponding at fig. and $h = 0.68, \Omega_b = 0.047, \Omega_m = 1, \Omega_\Lambda = 0$ for the one of fig. .
In a qualitative manner the resulting effect is the following: al low frequencies it is evident the Bremsstrahlung action, able to fill up until to $\lambda \gtrsim 100$ cm; the depression at intermediate frequencies is dued to comptonization; at high frequencies the Compton scattering generates a photons excess. The spectra corresponding at intermediate epochs show simply how the physical processes act on the CMBR spectrum.

6 Contribution of $\Lambda \neq 0$ to the spectral distortions

Once seen the cosmological parameter obtained by WMAP, one could stop itself on the costraints imposed to the cosmological constant values. The universe seems to be dominated, for its 70%, by a non null component of Λ, that however isn't an exhaustively answer to the problem related at the observation of this new acceleration in the cosmic expansion.
Many works duscuss about the effects of Λ on anisotropies of the CMBR. Here, we try to describe some effect that Λ could have on the spectral distortions, under the light of the simulations done.

6.1 Primordial epochs

We have seen that introducing Λ in the expression that drive the universe expansion it doesn't show any signicative change on spectral distortions, for energy injection at $z \gg 10^2$. In cases in wich the initial condition is represented by a Bose-Einstein spectrum, the differences between the evolution with Λ and the one without Λ are so small that we cannot consider them object for a study. The same thing is true for the following epoch. The contribution in this epochs is almost null because Λ has an important role in the cosmic evolution only at low redshift.

6.2 Reionization

In this case the effects of the introduction of Λ are the most impressive. The cosmic epoch (and its duration) when the energy dissipation accurs is the crucial element in determining the distortion entity. In the period considered Λ plays a foundamental role in the evolution of the scale factor: that's because the cases correspondig to reionization simulations are the ones that feel the effects of a non zero cosmological constant.
The effects are well visible in fig. and . The depression in the CMBR spectrum is more stressed when we consider an universe Λ dominated ($\Lambda = 0.73$). The difference, in terms of brightness temperature, from the initial condition (a plackian spectrum) amounts to $\Delta T_{br} \simeq 3 \times 10^{-5}$ (fig.). Instead, in the case when $\Lambda = 0$ the depression measures $\Delta T_{br} \simeq 1.75 \times 10^{-5}$. So, if we start two integration from the same redshift we have anyway two different behaviour and that it's dued to considering (or not) the cosmological constant in the scale factor evolution. In facts, we register a greater value of the comptonization parameter y in the case where Λ has a non zero value. This fact is related to a greater ezpansion time and we can deduce this also through the same definition of y:

$$y = \int_0^z \frac{\mathrm{d}z}{z} \frac{t_{exp}}{t_C}\,, \tag{4}$$

where t_{exp} is the expansion time and t_C is the time scale to reach kinetic equilibrium between matter and radiation. A greater expansion time means that, with parity in redshift interval for activation of the dissipation process, Compton scattering and Bremsstrahlung (processes related to the heating of the intergalactic medium) have much more time to distort the spectrum, creating a greater deviation from the Planckian spectrum.

7 Conclusion

Acknowledgements – Some of the calculations presented here have been carried out on an alpha digital unix machine at the IFP/CNR in Milano by using some NAG integration codes. C.B. warmly thanks L. Danese, G. De Zotti, and R. Salvaterra for constructive discussions and collaborations.

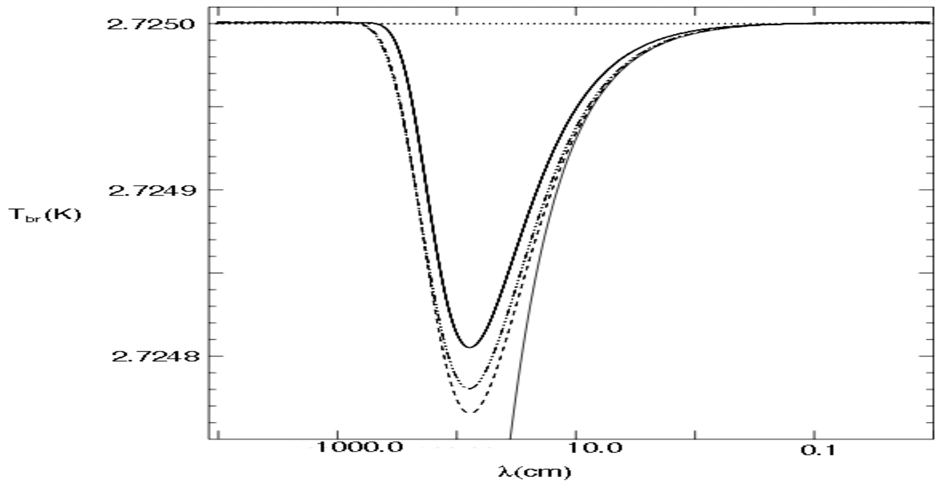

Figure 1: evolution of the CMBR spectrum starting from a BE-like spectrum. The flat dotted line represent a Planckian spectrum at $T_0 = 2.725°K$. The solid line wich get down quickly between 10 and 100 cm corresponds to the initial Bose-Einstein spectrum. The thick line is the final spectrum obtained by the numerical integration started from $y(z) \approx 5$. The dash-dotted line represents the spectrum at $y \simeq 2.1$, while the dashed line at $y \simeq 1.3$. Here is considered a ΛCDM model, with cosmological parameters given by $h = 0.68, \Omega_m = 0.29, \Omega_b = 0.047, \Omega_\Lambda = 0.73$

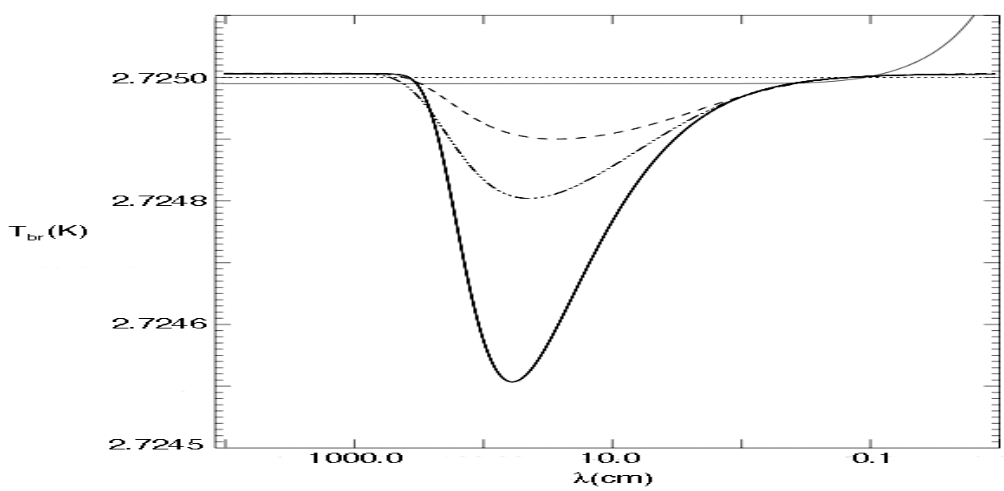

Figure 2: evolution of the CMBR spectrum starting from a superposition of blackbodies. The integration starts from $z = 4.35 \times 10^5$ at wich corresponds $y \simeq 2.2$. The flat line is the Planckian spectrum at $2.725°K$, the solid thick line represents the final spectrum. The intermediate lines correspond to the spectrum at $y \simeq 0.9$ (upper line) and $y \simeq 1.3$. The other solid line is the initial condition.

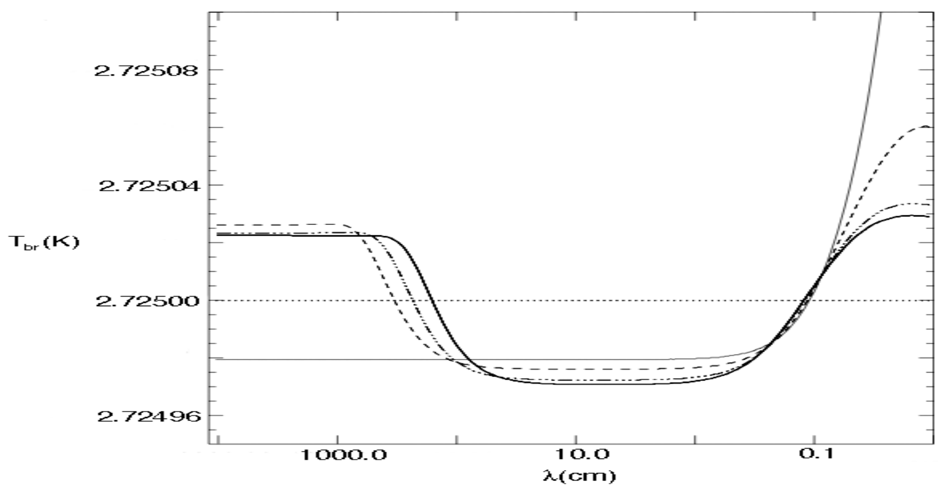

Figure 3: evolution of the CMBR spectrum starting from a superposition of blackbodies. The flat line is the Planckian spectrum at $T_0 = 2.725°K$. The integration started from $y \simeq 2 \times 10^{-2}$ (solid line), the dash-dotted line represents the spectrum at $y \simeq 6 \times 10^{-2}$ and the dashed line at $y \simeq 3 \times 10^{-2}$. The input cosmological parameters are: $\Delta\epsilon/\epsilon_i = 10^{-5}; h = 0.68, \Omega_m = 1, \Omega_b = 0.047, \Omega_\Lambda = 0$.

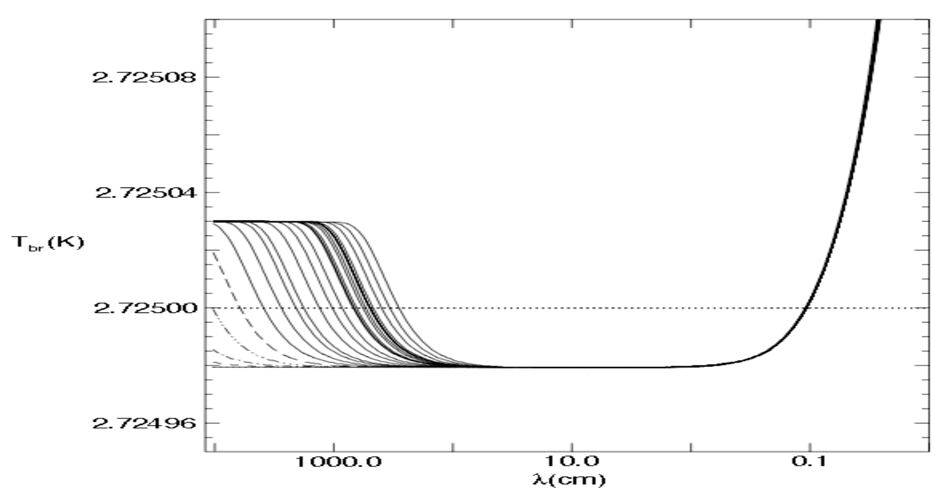

Figure 4: example of spectral evolution starting from a superposition of blackbodies. We focused the evolution on dependance from y. The filling up at lower frequencies is dued to the Bremsstrahlung effect, while the spectrum at higher frequencies is pratically an equilibrium one (superposition of blackbodies).

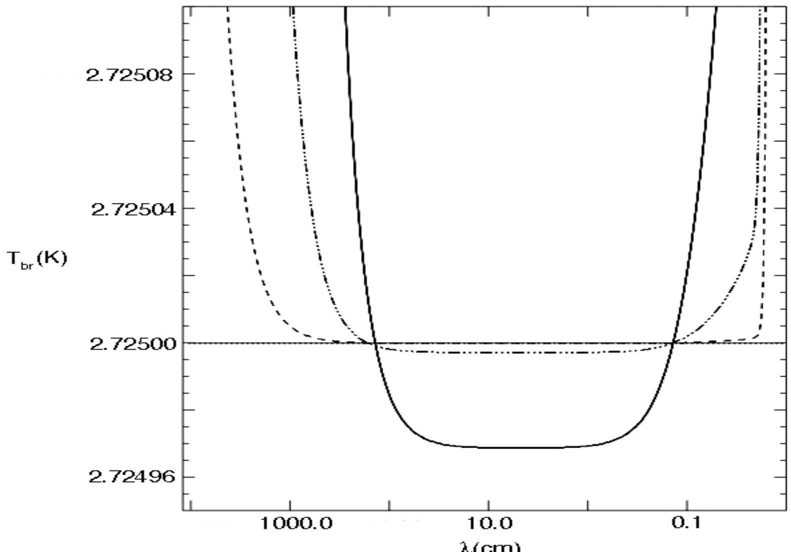

Figure 5: effect of the reionization on the CMBR spectrum. The initial condition is represented by a Planckian spectrum at $2.725°K$ (flat line). The distorted spectrum, after the reionization, is represented the solid thick line. In this case the integration started from $z \approx 20$ and the input cosmological parameters are $\Delta\epsilon/\epsilon_i = 10^{-5}; h = 0.68, \Omega_m = 1, \Omega_b = 0.047, \Omega_\Lambda = 0.73$.

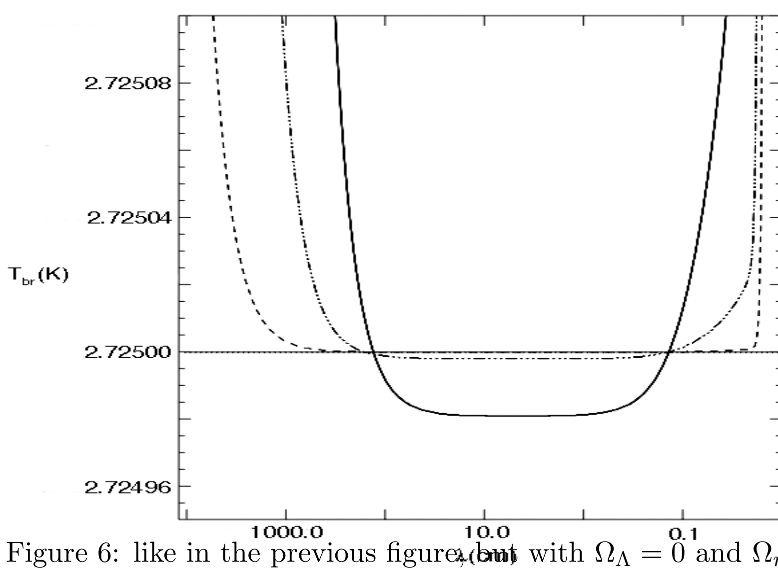

Figure 6: like in the previous figure, but with $\Omega_\Lambda = 0$ and $\Omega_m = 1$.

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