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ALIGNMENT OF LOW MULTIPOLES AND DIPOLE STRAYLIGHT CONTAMINATION: A POSSIBLE CONNECTION

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SUMMARY- The aim of this Report is to study the interplay between Dipole Straylight Contamination (DSC) and alignment of vectors associated to low multipoles. In particular we analyze the alignment between the dipole and (the vector associated to) quadrupole. We have shown that, depending on the specific realization of the random extraction of a_{2m}^{sky}, the effect of the DSC can be either a strong improvement either a strong decrease of the alignment, or only a very small effect. A statistical analysis is needed in order to provide conclusions on the kind of the effect of the connection between DSC and alignment.

1 Introduction

The anisotropy pattern of the Cosmic Microwave Background (CMB), obtained by Wilkinson Microwave Anisotropy Probe (WMAP), offers the possibility to test cosmological models with unprecedented precision [1].

Although WMAP data are largely consistent with the Standard Cosmological Model, there are some interesting deviations from it. In particular on the largest angular scales there are the so called "low ℓ anomalies". The first of these is the surprisingly low amplitude of the quadrupole (and of the octupole). The second of these is the unlikely (for a statistically isotropic random field) alignment of the quadrupole and the octupole. Moreover both quadrupole and octupole align with the CMB dipole [3, 4, 5]. The alignment anomaly has got the nickname of "Axis Of Evil" [2, 3, 4, 5, 6].

The measurement of this low ℓ pattern is affected by cosmic variance, foregrounds and systematics (see, e.g. [7, 8, 9] for a discussion on straylight contamination in the context of Planck Low Frequency Instrument (LFI) [10] and [11] in the context of WMAP).

In a simple analytical model [12], we tackled the systematic effect induced at low multipoles by the CMB kinematic dipole signal entering the main spillover. That analysis has been generalized in [13] relaxing the assumption of parallelism between the directions of the main spillover and of the spin axis (albeit keeping the main spillover center in the plane defined by the telescope axis and the spin axis).

The aim of this note is to investigate the implications of a non proper subtraction of the straylight contamination from the dipole on the low ℓ anomalies for Planck like spinning space experiments. In particular, we will focus on the alignment between quadrupole and dipole itself.

2 Multipole Vectors

In order to understand what is meant by alignment of multipoles it is necessary to introduce a new representation of cosmic microwave anisotropy maps along the lines of [5]. Instead of using the $a_{\ell m}$ representation of spherical harmonics it is possible to consider vectors. In particular, each multipole order ℓ is represented by ℓ unit vectors and one amplitude A

$$\sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\theta, \phi) \to A, \vec{u}_1, \cdots, \vec{u}_\ell \,. \tag{2.1}$$

From these vectors one could construct scalar quantities that are invariant under rotation (and then independent of the frame adopted for the computation). It is not equally easy to obtain scalar quantities directly from the $a_{\ell m}$ coefficients that, of course, depend on the coordinate system.

In order to understand Eq. (2.1) we start from the following observation: if f is solution of Laplace equation

$$\nabla^2 f = 0 \,, \tag{2.2}$$

where $\nabla^2 = \partial_x^2 + \partial_y^2 + \partial_z^2$ in cartesian coordinates, then it is possible to build a new solution f' applying a directional derivative to f

$$\nabla_{\vec{u}} f \equiv \vec{u} \cdot \nabla f = f' \,, \qquad \nabla^2 f' = 0 \,, \tag{2.3}$$

with the gradient $\nabla = (\partial_x, \partial_y, \partial_z)$. This happens because the two operators ∇^2 and $\nabla_{\vec{u}}$ commute [1]. Maxwell repeated this observation ℓ times considering the $1/r$ potential as starting solution. In this way, one obtains

$$f_\ell(x, y, z) = \nabla_{\vec{u}_\ell} \cdots \nabla_{\vec{u}_2} \nabla_{\vec{u}_1} \frac{1}{r}, \tag{2.4}$$

where $r = \sqrt{x^2 + y^2 + z^2}$. Observe the simple pattern that emerges as we apply the directional derivatives one at a time,

$$\begin{aligned}
f_0 &= \frac{1}{r} \\
f_1 &= \nabla_{\vec{u}_1} f_0 = \frac{(-1)(\vec{u}_1 \cdot \vec{r})}{r^3} \\
f_2 &= \nabla_{\vec{u}_2} f_1 = \frac{(3 \cdot 1)(\vec{u}_1 \cdot \vec{r})(\vec{u}_2 \cdot \vec{r}) + r^2(-\vec{u}_1 \cdot \vec{u}_2)}{r^5} \\
f_3 &= \nabla_{\vec{u}_3} f_2 = \frac{(-5 \cdot 3 \cdot 1)(\vec{u}_1 \cdot \vec{r})(\vec{u}_2 \cdot \vec{r})(\vec{u}_3 \cdot \vec{r}) + r^2(...)}{r^7}
\end{aligned} \tag{2.5}$$

where $\vec{r} = (x, y, z)$ while $r = \sqrt{\vec{r} \cdot \vec{r}}$ as before. The ellipsis $(\ldots)$ stands for a polynomial that we do not write explicitely, being unuseful for the currents purposes.

Moreover, writing f_ℓ in spherical coordinates once r is set to 1 one finds the following property

$$\tilde{\nabla}^2 f_\ell(1, \theta, \phi) = \ell(\ell + 1) f_\ell(1, \theta, \phi) \,, \tag{2.6}$$

[1] $\vec{u}$ does not depend on the coordinates (i.e. it is a constant vector).

where $\tilde{\nabla}^2$ is the angular Laplace operator defined as

$$\tilde{\nabla}^2 = -\left[\frac{1}{\sin\theta}\,\partial_\theta\left(\sin\theta\,\partial_\theta\right) + \frac{1}{\sin^2\theta}\,\partial^2_\phi\right]. \tag{2.7}$$

In other words $f_\ell(1,\theta,\phi)$ is eigenfunction of the angular part of the Laplace operator with eigenvalue given by $\ell(\ell+1)$. This is nothing but the definition of Spherical Harmonics $Y_{\ell,m}$. Therefore, we can write

$$Af_\ell(1,\theta,\phi) = \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\theta,\phi)\,, \tag{2.8}$$

where the amplitude A has been inserted because of normalization. Eq. (2.8) makes more clear the association represented by Eq. (2.1). In order to fully understand this association we count the equations and the unknowns that we have to deal with. From Eq. (2.8) we have $2\ell+1$ equations (one equation for each $a_{\ell m}$ [2]) plus ℓ equations from the normality conditions of the vectors (i.e. $\vec{u}_i \cdot \vec{u}_i = 1$ where i runs from 1 to ℓ). Therefore the total number of independent equations is $3\ell+1$. This is also the number of unknowns because we have 3 unkowns for each vector plus 1 given by the amplitude A. The system is then solvable.

We will see that it is possible to find an analytical solution only for $\ell = 1$ while already for $\ell = 2$ numerical methods are needed.

3 Dipole and Quadrupole

For $\ell = 1$ we have

$$-A(\vec{d}\cdot\vec{r}) = \sum_{m=-1}^{1} a_{1m} Y_{1m}(\theta,\phi)\,, \tag{3.1}$$

$$\vec{d}\cdot\vec{d} = 1 \tag{3.2}$$

and considering $Y_{1m}(\theta,\phi)$ given in [14] we have the following analytic solution

$$d_x = \pm a^{(R)}_{11}\Big/\sqrt{a^2_{10}/2 + ((a^{(R)}_{11})^2 + (a^{(I)}_{11})^2)}\,, \tag{3.3}$$

$$d_y = \pm a^{(I)}_{11}\Big/\sqrt{a^2_{10}/2 + ((a^{(R)}_{11})^2 + (a^{(I)}_{11})^2)}\,, \tag{3.4}$$

$$d_z = \pm a_{10}\Big/\sqrt{a^2_{10} + 2((a^{(R)}_{11})^2 + (a^{(I)}_{11})^2)}\,, \tag{3.5}$$

$$A = \mp\frac{1}{2}\sqrt{\frac{3}{\pi}}\sqrt{a^2_{10} + 2((a^{(R)}_{11})^2 + (a^{(I)}_{11})^2)} \tag{3.6}$$

where $\vec{d} = (d_x, d_y, d_z)$ and the labels (R) and (I) stand for real and imaginary part.

For $\ell = 2$ we have

$$-A(3(\vec{u}\cdot\vec{r})(\vec{w}\cdot\vec{r}) + r^2(-\vec{u}\cdot\vec{w})) = \sum_{m=-2}^{2} a_{2m} Y_{2m}(\theta,\phi)\,, \tag{3.7}$$

$$\vec{u}\cdot\vec{u} = 1\,, \tag{3.8}$$

$$\vec{w}\cdot\vec{w} = 1 \tag{3.9}$$

[2] In fact we would have $4\ell+1$ equation because each ℓ different from 0 has a real and imaginary part. But considering that $a_{\ell m}$ with $m > 0$ are related to those with $m < 0$) through $a^\star_{\ell m} = (-1)^m a_{\ell -m}$ we have $2\ell+1$ equations.

and considering $Y_{2m}(\theta,\phi)$ given in [14] we have the following set of equations

$$-\frac{1}{2}(u_x w_x + u_y w_y - 2u_z w_z)A = a_{20}\sqrt{\frac{5}{16\pi}}\,, \tag{3.10}$$

$$\frac{3}{2}(u_x w_x - u_y w_y)A = 2a_{22}^{(R)}\sqrt{\frac{15}{32\pi}}\,, \tag{3.11}$$

$$\frac{3}{2}(u_z w_x + u_x w_z)A = -a_{21}^{(R)}\sqrt{\frac{15}{8\pi}}\,, \tag{3.12}$$

$$\frac{3}{2}(u_z w_y + u_y w_z)A = a_{21}^{(I)}\sqrt{\frac{15}{8\pi}}\,, \tag{3.13}$$

$$\frac{3}{2}(u_y w_x + u_x w_y)A = -a_{22}^{(I)}\sqrt{\frac{15}{32\pi}}\,, \tag{3.14}$$

$$u_x^2 + u_y^2 + u_z^2 = 1\,, \tag{3.15}$$

$$w_x^2 + w_y^2 + w_z^2 = 1\,, \tag{3.16}$$

where $\vec{u} = (u_x, u_y, u_z)$ and $\vec{w} = (w_x, w_y, w_z)$. As already said in the previous section, this set of equations (from Eq. (3.10) to Eq. (3.16)) is too complicated to be solved analytically. We will numerically obtain $\vec{u}$, $\vec{w}$, and A.

4 Analysis

The analysis is perfomed in the following way. We take a_{1m} as in [12] and extract randomly a_{2m}^{sky} such that $C_2 \simeq 10^3 \mu K^2$. Then we compute $\vec{d}$ and $\vec{u}$ and $\vec{w}$. The vectors $\vec{u}$ and $\vec{w}$ define a plane. We call $\vec{q}$ the vector orthogonal to that plane:

$$\vec{q} = \vec{u} \times \vec{w}\,. \tag{4.1}$$

Since we are intersted in alignment we normalize it

$$\vec{q} \to \vec{q}/\sqrt{\vec{q}\cdot\vec{q}}\,, \tag{4.2}$$

and then compute the scalar product SP with $\vec{d}$ and consider it as an estimator of alignment

$$SP_{sky} = \vec{d}\cdot\vec{q}\,. \tag{4.3}$$

The same procedure is repeated considering the systematic effect of dipole straylight contamination computed in [12]. This means that instead of plugging a_{2m}^{sky} in the set of equations we plug $a_{2m}^{sky} + a_{2m}^{SL}$ where the non vanishing a_{2m}^{SL} are

$$a_{22}^{SL(R)} = -p\frac{\sqrt{5}}{3\pi}\left(1 + \frac{\cos\Delta\sin\Delta}{\Delta}\right)a_{11}^{(R)} \tag{4.4}$$

$$a_{22}^{SL(I)} = -2p\frac{\sqrt{5}}{3\pi}\left(1 + \frac{\cos\Delta\sin\Delta}{\Delta}\right)a_{11}^{(I)}\,. \tag{4.5}$$

We compute SP_{total}. It can be compared with SP_{sky} for various values of the parameter we used to describe the dipole straylight contamination, i.e. p and Δ [3]). In fact, SP_{sky} can be obtained by SP_{total} when $p = 0$ (see the figures in the next section). If $SP_{total}(p)$ is larger (smaller) than $SP_{total}(p = 0) = SP_{sky}$, an improvement (destruction) of the alignment is obtained.

[3] See [12] for definitions of these parameters.

5 Results

The results of this analysis are plotted in Figs. 1, 2, and 3. In all cases we have taken $\Delta = \pi/10$. We note that, depending on the random realization of a_{2m}^{sky}, the result of the dipole straylight contamination can produce an improvement of alignment, a destruction of the alignment, or only a very small effect.

Notice that all the left panels of Figs. 1, 2, and 3 contain the plot of SP_{total} also for negative values of p. In fact this is equivalent to SP_{total} with p positive and $a_{2m}^{sky} \to -a_{2m}^{sky}$ [4].

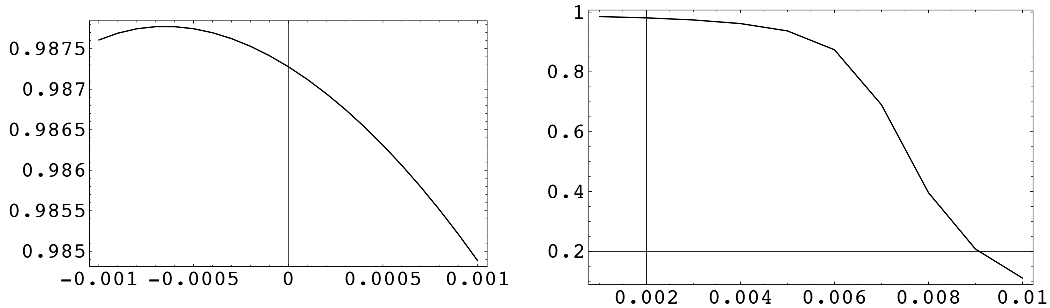

Figure 1: Left Panel: SP_{total} versus p. Right Panel: The same as the Left Panel but for larger and positive p. In both panels we have considered $a_{\ell m}^{sky}$ as the ones given by WMAP.

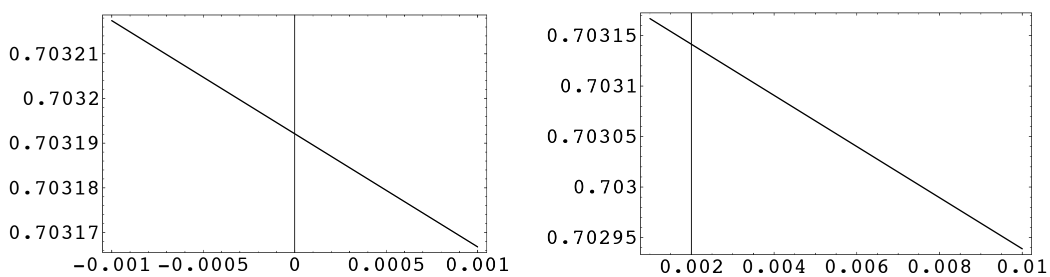

Figure 2: Left Panel: SP_{total} versus p. Right Panel: The same as the Left Panel but for larger and positive p. In both panels we have considered $a_{\ell m}^{sky}$ randomly chosen such that $C_2 \simeq 10^3 \mu K^2$.

6 Conclusion

The aim of this Report is to show a possible connection between the straylight systematic effect introduced by the CMB dipole (DSC), as discussed in [12, 13], and the anomaly of alignment of low multipoles.

We have showed that there is a strong interplay between the DSC and the Dipole-Quadrupole alignment. Depending on the specific realization of the random extraction of

[4] To be more precise if p is negative then a_{2m}^{SL} change sign. But with the redefinition $A \to -A$ this corresponds to the change of sign of a_{2m}^{sky}. SP_{total} is insensitive to the change $A \to -A$ because just its absolute value is considered.

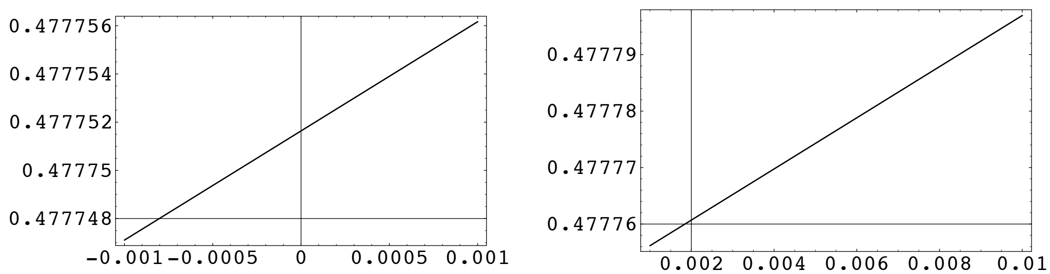

Figure 3: Left Panel: SP_{total} versus p. Right Panel: The same as the Left Panel but for larger and positive p. In both panels we have considered $a_{\ell m}^{sky}$ randomly chosen such that $C_2 \simeq 10^3 \mu K^2$ (but for a different realization with respect to Fig. 2).

a_{2m}^{sky}, the effect of the DSC can be either a strong improvement either a strong decrease of the alignment, or only a very small effect.

A deeper study with an appropriate statistical analysis is needed to draw more precise conclusions that could be applied to the WMAP data and to the PLANCK mission.

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