The angular response of a rectangular mechanical collimator. The case of the HXMT collimators

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1 Goal of the report

In this report we present the general theory to compute the angular response of a rectangular mechanical collimator. We find the analytical dependence on both the azimuthal angle, θ, and the polar angle, φ. Then we apply the general formalism to the case of the collimators aboard the Hard X–ray Telescope (HXMT).

2 Introduction

The high energy (HE) instrument aboard the Chinese Hard X–ray Modulation Telescope (HXMT) consists of 18 cylindrical detectors, each formed by a phoswich module and its collimator. Each detector unit is a NaI(Tl)/CsI(Na) phoswich scintillation detector with a diameter of 19 cm.

Because the image reconstruction is based on the so-called *Direct Demodulation* technique [2], the collimator field of view (FOV) must be asymmetric. According to [1], the collimators defines a FOV of $5.71^\circ \times 1.12^\circ$ (that we propose to increase to $5.86^\circ \times 1.12^\circ$ [3]). The directions of the long axes of the 18 FOVs are different, varying with a step size of 10° (see Figure 1). The angular response of each collimator is then combined together in order to reconstruct the position of a source.

We will begin by computing the azimuthal and polar dependence of the angular response of a rectangular mechanical collimator. Then we will combine together the angular responses of all the 18 collimators in the geometrical configuration as shown in Figure 1 to compute the cumulative HXMT angular response.

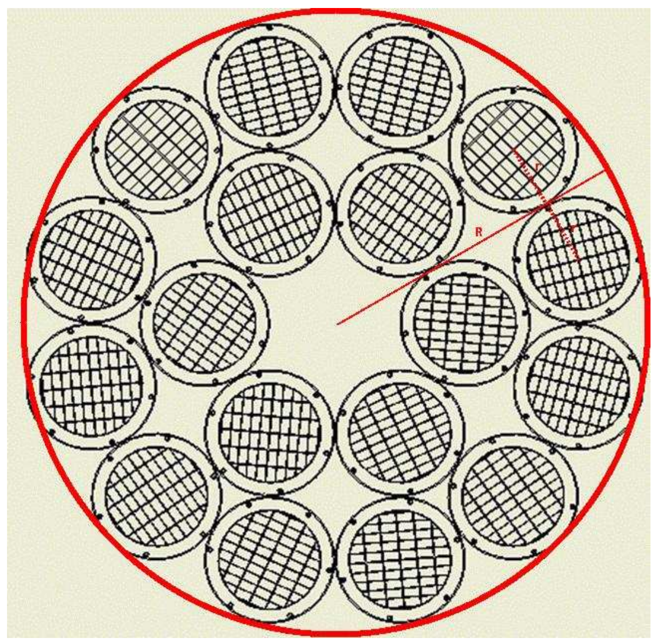

Figure 1: The FOVs of the 18 mechanical collimators aboard HXMT.

In the following we will assume that the collimator walls are perfectly opaque to the X–ray radiation. The case of X–ray transmission through the collimator walls is treated into detail in [3]. The collimator cell dimensions are: $a = 30.8$ mm, $b = 6$ mm, $h = 300$ mm.

3 Angular response: θ and φ dependence

In Figure 2 we show a sketch of a collimator cell, with the shadows of its walls cast on its interior (see Figure 3 for a view from above). Our goal is to find a relation between the two angles θ and φ and the area of the illuminated part of the detector, that is the rectangle $KBHD''$, shown in red in Figure 2.

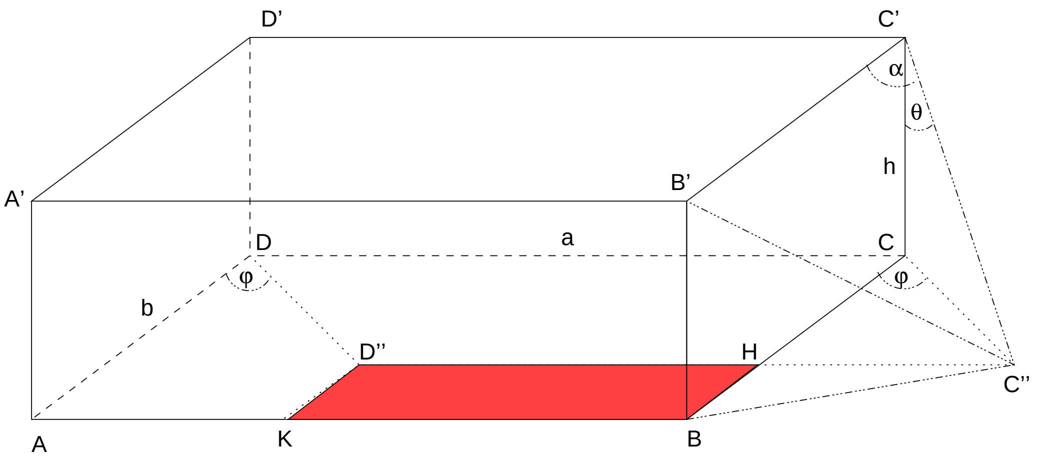

Figure 2: Schematic view of the collimator cell, together with the viewing angles θ, α and φ. In red the area left illuminated by the shadows cast by the collimator walls.

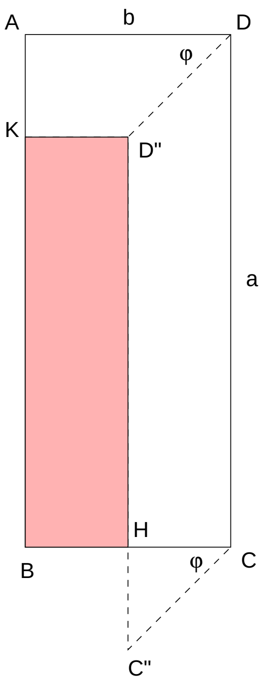

Figure 3: Schematic view of the collimator cell: top view.

First, let us start with the right triangle $CC'C''$, for which:

$$C'C'' = \frac{h}{\cos\theta} \qquad\qquad CC'' = h\,\tan\theta \tag{1}$$

Then the triangle $B'C'C''$ for which, by applying the Carnot theorem, we can write

$$B'C''^2 = b^2 + \frac{h^2}{\cos^2\theta} - \frac{2bh}{\cos\theta}\,\cos\alpha \tag{2}$$

Then the right triangle $BB'C''$ for which, by applying the Pythagorean theorem, we can write

$$\begin{aligned} BC''^2 &= B'C''^2 - h^2 \\ &= b^2 + \frac{h^2}{\cos^2\theta} - \frac{2bh}{\cos\theta}\,\cos\alpha - h^2 \\ &= b^2 + h^2\left(\frac{1}{\cos^2\theta} - 1\right) - \frac{2bh}{\cos\theta}\,\cos\alpha - h^2 \\ &= b^2 + h^2\tan^2\theta - \frac{2bh}{\cos\theta}\,\cos\alpha \end{aligned} \tag{3}$$

Finally, by applying the Carnot theorem to the triangle BCC'' we have

$$BC''^2 = b^2 + h^2\tan^2\theta - 2\,b\,h\,\tan\theta\,\cos\varphi \tag{4}$$

where φ is the angle BCC'', that is the angle measured on the *detector plane*. By comparing Eq. (3) and (4) we obtain

$$b^2 + h^2 \tan^2\theta - \frac{2bh}{\cos\theta}\cos\alpha = b^2 + h^2\tan^2\theta - 2\,b\,h\,\tan\theta\,\cos\varphi \tag{5}$$

from which the requested relation between the three angles θ, φ and α:

$$\cos\varphi = \frac{\cos\alpha}{\sin\theta} \tag{6}$$

Two important quantities necessary to compute the area of the rectangle $KBHD''$ are HC and HC'', which are

$$HC'' = h\,\tan\theta\,\sin\varphi \qquad\qquad HC = h\,\tan\theta\,\cos\varphi \tag{7}$$

We now have all the elements to compute the area of $KBHD''$ in terms of the angles θ and φ. Indeed

$$\begin{aligned} A_{KBHD''} &= KB \times BH \\ &= (a - AK)(b - HC) = (a - HC'')(b - HC) \\ &= (a - h\,\tan\theta\,\sin\varphi)(b - h\,\tan\theta\,\cos\varphi) \end{aligned} \tag{8}$$

The normalized (that is, divided by ab) illuminated area, or the collimator angular response, is

$$\mathcal{R}(\theta,\varphi) = \left[1 - \frac{h}{a}\,\tan\theta\,\sin\varphi\right]\left[1 - \frac{h}{b}\,\tan\theta\,\cos\varphi\right] \tag{9}$$

In Figure 4 we show a plot of $\mathcal{R}(\theta,\varphi)$ given the cell dimensions for the HXMT collimator geometry.

3.1 Constraints on θ and φ

In order to have the detector illuminated by the X–rays it is necessary that the area of the rectangle $KBHD''$ be not zero, that is

$$\begin{aligned} 0 \le HC'' \le a \quad &\Rightarrow \quad 0 \le h\,\tan\theta\,\sin\varphi \le a \\ 0 \le HC \le b \quad &\Rightarrow \quad 0 \le h\,\tan\theta\,\cos\varphi \le b \end{aligned} \tag{10}$$

A constraint on the angle θ is obtained by noticing that its maximum value $\theta_{\max}$ is achieved in correspondence of the diagonal of the collimator BD', that is

$$\tan\theta_{\max} = \frac{\sqrt{a^2+b^2}}{h} \tag{11}$$

In Figure 5 we show the above mentioned constraints, in a φ–θ plane, for the HXMT collimator cell dimensions.

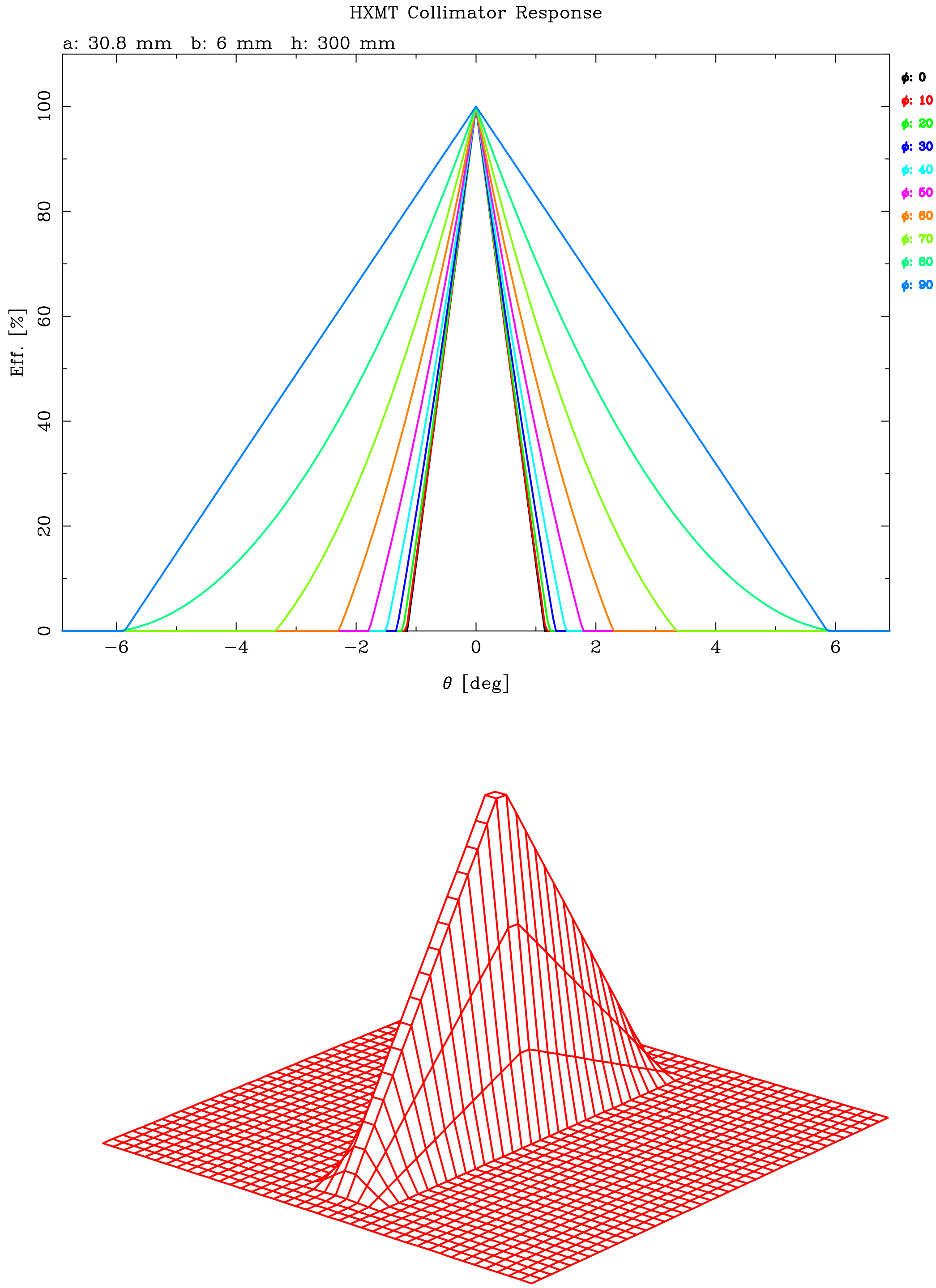

Figure 4: The HXMT collimator response function $\mathcal{R}(\theta, \varphi)$. *Top:* $\mathcal{R}$ for different values of the φ angle. *Bottom:* 3-dimensional plot in polar coordinates.

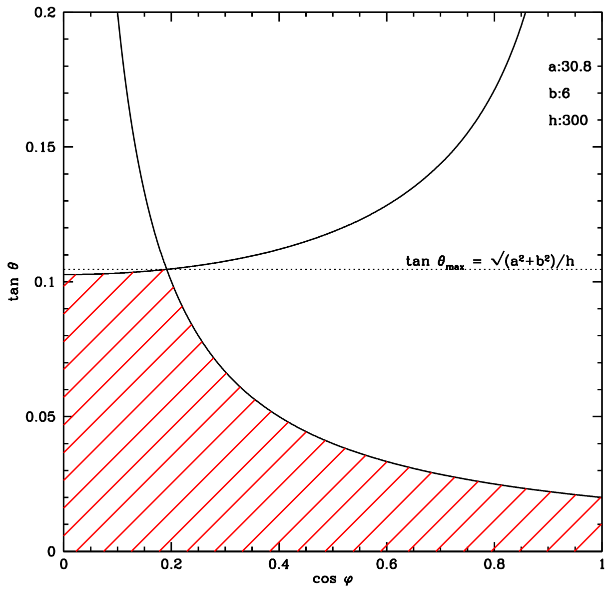

Figure 5: Constraints on the θ and φ angles in order to have an illuminated area in the detector plane. We used the HXMT collimator cell dimensions $a = 30.8$ mm, $b = 6$ mm, $h = 300$ mm. In this case $\theta_{\max} = 5.971°$.

4 The HXMT collimator response for 18 collimators

By means of Eq. (9) it is now possible to build the HXMT total angular response function (a sort of a *"Point Spread Function"*) by summing up all the 18 collimator angular responses. The result is shown in Figure 6, where in red are marked the Full Width at Zero Intensity (FWZI) and the Full Width at Half Maximum (the latter being equal to 1.60°).

The angular response integrated in φ, that is a slice along a $\varphi =$ constant direction, is shown in Figure 7. The shape does not resemble a Gaussian at all, but it is more similar to a Lorentzian, as is evident from Figure 8, where the two fits are compared. In Table 1 we summarize the fit results.

Acknowledgements

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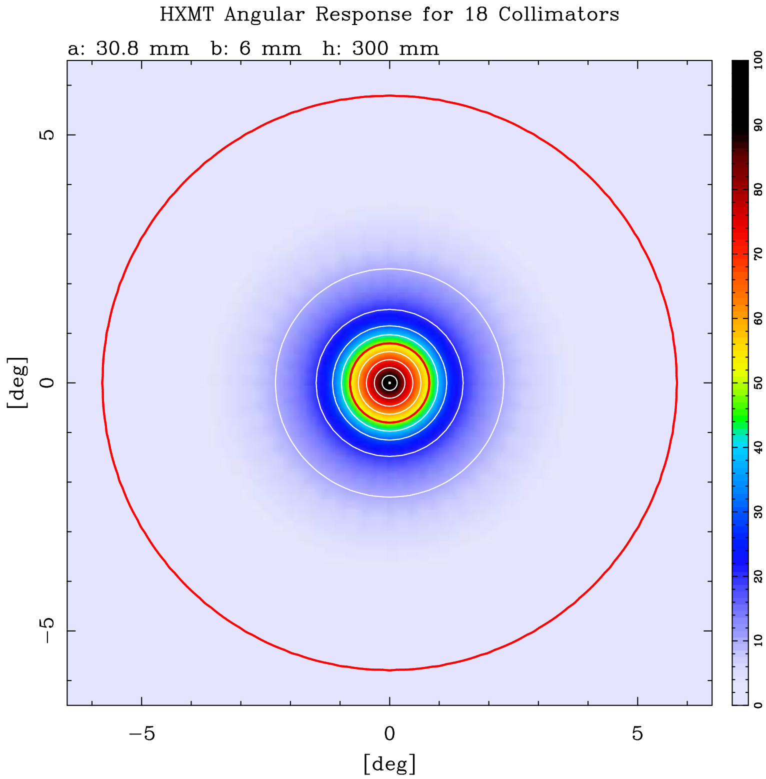

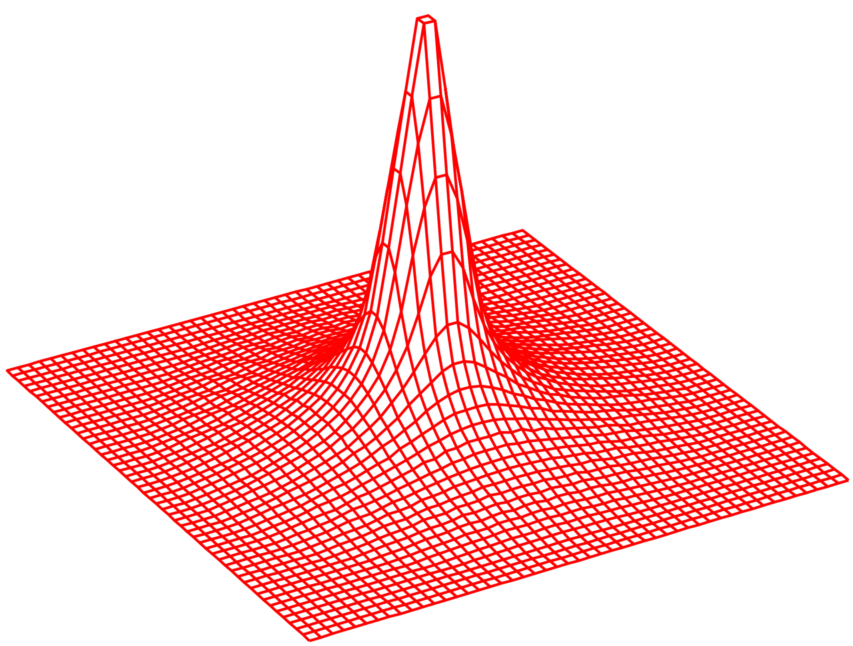

Figure 6: The HXMT angular response integrated for all the 18 HXMT collimators. *Top:* contour plot (in red the FWZI and FWHM; the latter being 1.60°). *Bottom:* 3-dimensional plot in polar coordinates.

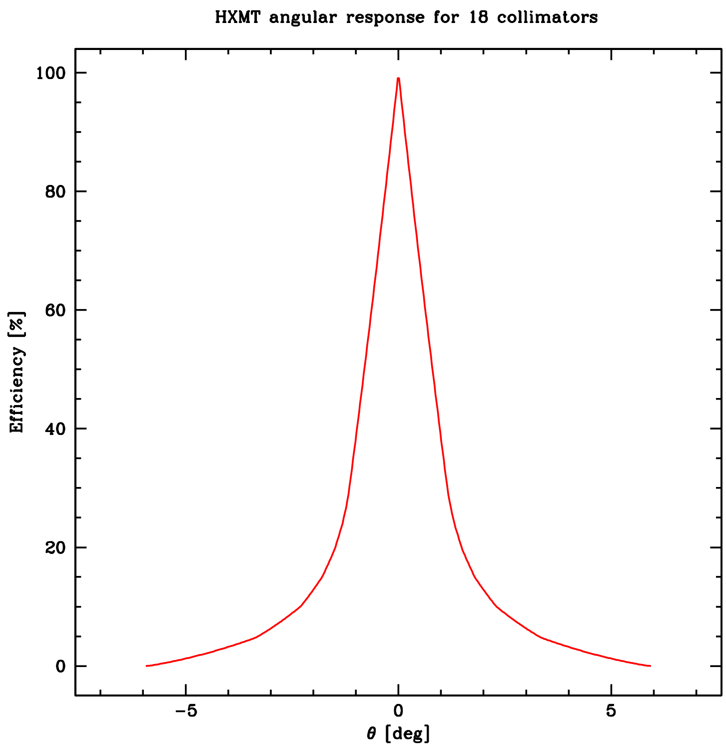

Figure 7: The HXMT angular response integrated in φ. It has been obtained by slicing Figure 6 along a $\varphi =$ constant direction. The measured FWHM is 1.60°.

Table 1: Best fit parameters for the Gaussian and Lorentzian fit to the HXMT angular response integrated in φ. The measured FWHM is 1.60°, while from the Gaussian fit it results 2.039°.

Parameter	Gaussian	Lorentzian
GC	7.5×10^{-10}	
GW	0.866	
GN	84.58	
LC		2.5×10^{-10}
LW		1.595
LN		94.72
χ^2/dof (497 dof)	19	1.6

$$\text{Gaussian} \equiv \text{GN} \exp\left[\frac{1}{2}\left(\frac{x-\text{GC}}{\text{GW}}\right)^2\right] \quad ; \quad \text{Lorentzian} \equiv \frac{\text{LN}}{1+\left(\frac{2(x-\text{LC})}{\text{LW}}\right)^2}$$

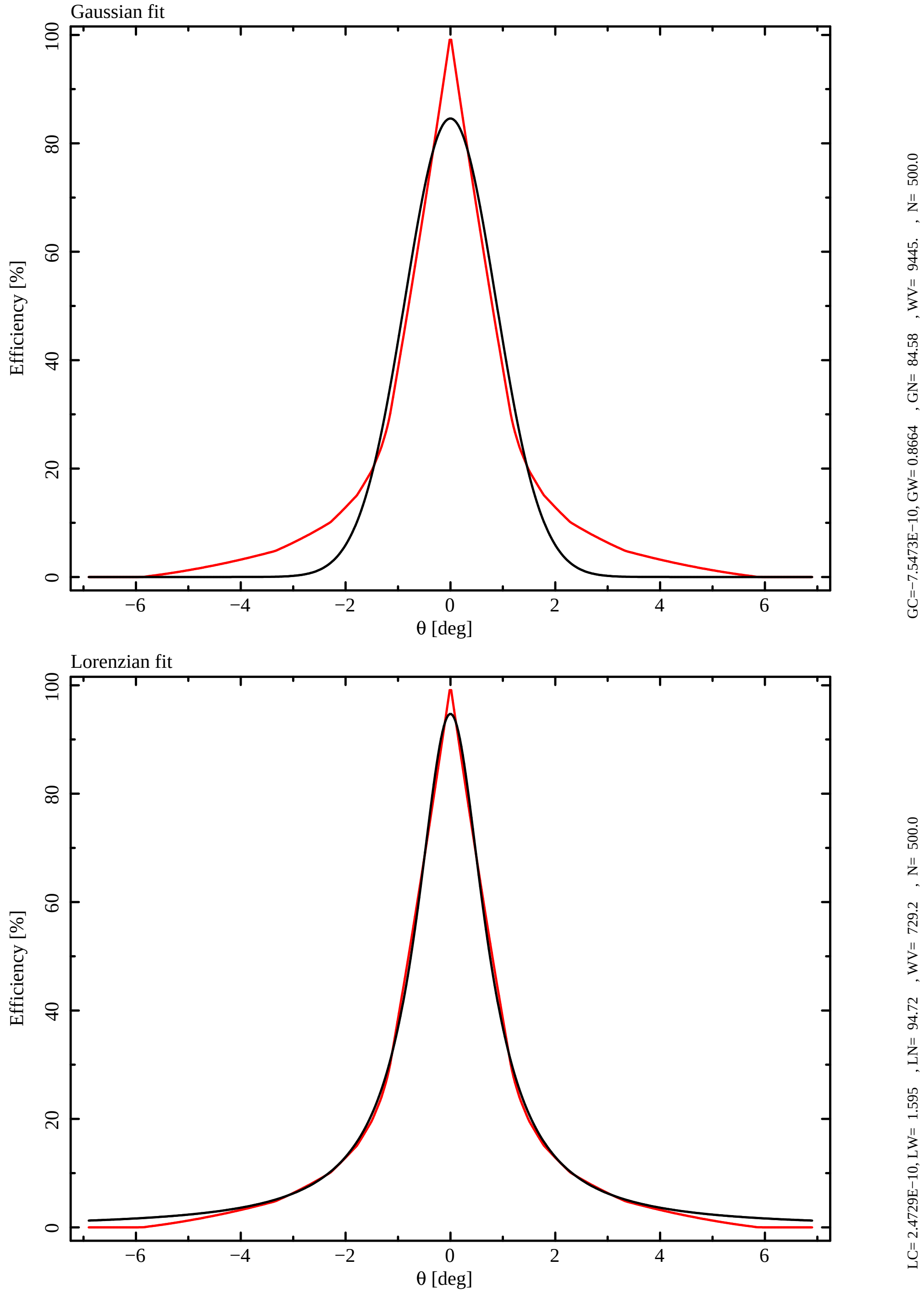

Figure 8: Gaussian (*Top*) and Lorentzian (*Bottom*) fit to the HXMT angular response integrated in φ shown in Figure 7. It is quite evident that the shape is better described in terms of a Lorentzian function. The best fit parameters are listed in Table 1.

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