Internal Report IASF-BO 621/2013

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March 29, 2013

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SUMMARY – We present a semi-analytical study aimed at the characterization of CMB free-free spectral distortions at relatively low redshift taking into account the amplification effect coming from clumping factor associated to the density contrast of the intergalactic medium. The analysis is carried out in the framework of a standard cosmological model for selected astrophysical and phenomenological reionization histories, adopting a standard semi-analytical description of the free-free distortion excess and an new semi-analytical description of the density contrast, based on numerical Boltzmann code results. We evaluate the impact of the clumping factor for the different scenarios and discuss the wavelength dependence of the signal.

1 Introduction

After the electron-pair annihilation and before the recombination epoch, the evolution of the photon occupation number is suitably described by the Kompaneets equation [5], an approximation of the generalized kinetic equation which, in its generalized form, includes, other than Compton scattering, that conserves the photon number, all the possible photon emission or absorption processes such as double Compton and bremsstrahlung. Compton and double Compton depend linearly on the baryon density in the Universe. The bremsstrahlung term is instead proportional to the square of the baryon density thus, to accurately describe this process, it should be accounted for a non homogeneous, evolving intergalactic medium (IGM). Indeed, the matter power spectrum depends on cosmological parameters and, obviously, it is a function on the wavenumber which is inversely proportional to the linear scale. The so-called matter transfer function, $T(k)$, defines the matter power spectrum evolution from a primordial time, usually identified at the end of the inflationary stage, to a desired final time, and is associated associated to the linear growth of the perturbations.

The IGM density can be expressed in terms of two contributions, a mean density term and a perturbative term such as:

$$\rho = \langle\rho\rangle(1+\delta) \; ; \tag{1}$$

by definition δ should have a null mean, $\langle\delta\rangle = 0$. On the other hand, its variance is, in principle, not vanishing, thereby it can imply an amplification of the free-free process with respect to the case of homogeneous distribution defined by the multiplicative factor [8]:

$$\frac{\langle\rho^2\rangle}{\langle\rho\rangle^2} = 1 + \langle\delta^2\rangle = 1 + \sigma^2 > 1\,. \tag{2}$$

In order to numerically estimate the free-free distortion taking int account this effect, we developed the Fortran90 code **FF_clump** based on analytic approximations of the clumping factor coming from a previous study [12], and tracking the effect induced on CMB spectral distortions for three different reionization scenarios, all of them described by a standard cosmological model, and at various cutoff wave numbers k_{max}.

In this report, we describe the code, the ionization histories adopted for the analysis, the corresponding clumping factor and its characteristic growth at remarkable low redshifts, during the achievement of an almost fully ionized Universe. Finally, we provide a precise characterization of the CMB temperature free-free spectral distortions induced, including a not null density contrast.

2 Free-free process

The interaction between matter and radiation during the early stages of Universe evolution and, extensively, the evolution of the photon occupation number (η) is described by the complete kinetic equation [4] which encloses all physical processes able to conserve the photon number, such as the Compton scattering, and all possible photon production (absorption) processes, as double Compton or free-free.

In the Kompaneets equation [5], an approximate solution of the Boltzmann equation in the Thomson limit for a Maxwellian electrons distribution, the bremsstrahlung term can be expressed by:

$$\left(\frac{\partial\eta}{\partial t}\right)_B = g_B(x_e)\frac{\exp(-x_e)}{x_e^3}K_B[1-\eta(\exp(x_e)-1)] \; , \tag{3}$$

where g_B is the Gaunt factor, provided in terms of the dimensionless frequency $x_e = h\nu/(k_B T_e)$ $= x/\phi$, being $\phi = T_e/T_r$ the electron photon temperature ratio. Since the medium is in a partial ionization state, the Gaunt factor at each redshift has been properly weighted accounting for the correct hydrogen and helium ionization fractions.

The coefficient K_B defines the rate at which bremsstrahlung changes the photon occupation number with regard to fundamental cosmological parameters, such as [1, 2, 3]:

$$K_B(z) = \frac{8\pi}{3}\frac{e^6 h^2 n_e(n_H + 4n_{He})}{m(6\pi m k T_e)^{1/2}(kT_e)^3} = K_{0B}(z)\phi^{-7/2}\,, \tag{4}$$

and

$$K_{0B} \simeq 2.6\cdot 10^{-25}\left(\frac{T_0}{2.7K}\right)^{-7/2}(1+z)^{5/2}\hat{\Omega}_b^2\ s^{-1}. \tag{5}$$

The total Gaunt factor appearing in Eq. (3), $g_B(x_e)$, has been evaluated following the approximation of [7] and [3] implemented as explained below. Indeed, the above equations, and in particular the free-free rate defined in Eq. (4), hold in a total ionized medium. Actually, being this analysis focused at intermediate and low redshifts, resulting into late spectral distortions, this condition is not reasonable at all since the plasma, during this period, stands in different ionization states. The initial heating redshift is set in fact in the code at $z_h = 30$. Precisely, the electrons, hydrogen and helium number densities become:

$$n_e(n_H + 4n_{He}) = n_e^F(n_{H^+} + 4n_{He^{++}} + n_{He^+})\,, \tag{6}$$

being the free electron fraction $n_e^F = \chi_e n_e$, with n_e the total electron fraction, given by:

$$n_e = \frac{1+f_H}{2}n_b\,, \tag{7}$$

and where n_b is the baryon number density, provided by:

$$n_b = \frac{\rho_b}{m_b} = 2.8\cdot 10^{-6}\hat{\Omega}_b(1+z)^3\,. \tag{8}$$

The ionization fraction, χ_e, is being determined by the reionization history. The repartition of atoms in different ionization states has been evaluated, at equilibrium, by solving the Saha equations [10], a system that describes the ratio between two different ionization states of an element. The weighted total Gaunt factor is:

$$g_B(x_e) = (\chi_{H^+} + \chi_{He^+})g_B^H(x_e) + \chi_{He^+}g_B^{He}(x_e)\,. \tag{9}$$

3 Distorted spectra

The photon occupation number at a time t, assuming an initial blackbody spectrum, $\eta_{BB,i}$, and under the only effect of bremsstrahlung emission, can be well represented by the relation:

$$\eta \simeq \eta_{BB,i} + \frac{y_B}{x^3} - u\frac{2}{x/\phi}\,, \tag{10}$$

with $u = u(t)$ the Comptonization parameter, and ϕ_i the initial electron temperature necessary for the evaluation of the distortion parameter, related to the fractional amount of energy exchanged between matter and radiation by $\phi_i = T_i/T_r = (1+\Delta\varepsilon/\varepsilon_i)^{-1/4} \simeq 1-u$ (where u is computed at the final epoch).

Finally, the free-free parameter $y_B(t)$ turns out to be:

$$y_B = \int_{t_h}^{t} (\phi - \phi_i)\phi^{-3/2} g_B(x,\phi) K_{0B} dt = \int_{z}^{z_h} (\phi - \phi_i)\phi^{-3/2} g_B(x,\phi) K_{0B} t_{exp} \frac{dz}{1+z} \,. \tag{11}$$

4 Expansion time

The cosmic evolution of the Universe is treated in the code with the time variable ω, a cosmic scale factor normalized when CMB temperature is $k_B T = m_e c^2$, specified by:

$$\omega = \frac{m_e c^2}{k_B T_r} = \frac{m_e c^2}{k_B T_0 (1+z)} \,. \tag{12}$$

In the integral of Eq. (11), the conversion factor between d/dz, or the corresponding d/dt, and $d/d\omega$ to be replaced is given by $d/dt = d/d\omega \cdot d\omega/dt = d/d\omega \cdot \dot{\omega}$.

Taking into account the recent cosmic acceleration of the Universe, parametrized by the cosmological constant Ω_Λ, and the curvature term Ω_K and , the complete expression for $\dot{\omega}$ becomes [6]:

$$\frac{1}{\dot{\omega}} = \frac{\tau_{g1}\omega}{\left[1 + \beta\omega \left(1 + \frac{(\Omega_K/\Omega_m)\omega}{2.164\cdot 10^9} + \frac{(\Omega_\Lambda/\Omega_m)\omega^3}{(2.164\cdot 10^9)^3}\right)\right]^{1/2}} \,, \tag{13}$$

where β is the initial ratio between matter and radiation energy densities and τ_{g1} can be seen as an initial gravitational time scale, defined as:

$$\beta = \frac{\rho_{m1}}{\rho_{r1}} = 3.5 \cdot 10^{-6} h_{50}^2 \Omega_{tot} (T_0/2.7\mathrm{K})^{-3} \,, \tag{14}$$

$$\frac{1}{\tau_{g1}} = \left(\frac{8\pi}{3} G \rho_{r1}\right)^{1/2} = \left[\frac{8\pi}{3} G \frac{a}{c^2} \left(\frac{m_e c^2}{k}\right)^4\right]^{1/2} = 0.076\ \mathrm{s}^{-1} \,. \tag{15}$$

The gravitational time scale term accounts for the relativistic neutrinos contribution to the Universe's dynamic, such as:

$$\tau_{g1} = \frac{13.11022}{(\kappa_\nu)^{1/2}} \,, \tag{16}$$

where κ_ν is the present ratio of neutrino to photon energy densities:

$$\kappa_\nu = \frac{1}{2}\left(g_\gamma + \frac{7}{8} g_\nu N_\nu (N_\nu^{eff})^{(-4/3)}\right) = 1 + \frac{7}{8} N_\nu \left(\frac{4}{11}\right)^{4/3} \,, \tag{17}$$

being g_γ the effective number of photons spin states, N_ν the number of relativistic, 2-component neutrino species, N_ν^{eff} the effective number of species at the decoupling epoch. Typically, for 3 species of massless neutrinos and for 2 massless photon spin states, $\kappa \simeq 1.68$.

5 Reionization histories

To investigate the effect triggered by the density contrast of the IGM on CMB spectral distortions, we focalized our analysis on three well determined Universe's reionization models, two astrophysical, namely suppression (S) and filtering (F), and one phenomenological, late double peaked (L) histories (see [9, 11, 13] for further details about the histories). The optical depth for S and F is intrinsically specified by the model itself, being $\tau_S = 0.1017$ and $\tau_F = 0.0631$, while for L model we balanced its main free parameters in order to derive comparable optical depths, namely setting $\tau_L = \tau_S$.

In Fig. (1), we directly compare the time evolution of the ionization fraction (top panel), of the electron temperature (top middle panel), of the bremsstrahlung rate (bottom middle panel) and, lastly, of the clumping factor (bottom panel) for S, F and L models. As emerges from the plots, the clumping factor gives an important contribution at low redshift, when the plasma is characterized by a high ionization fraction, as highlighted in the top panel of figure.

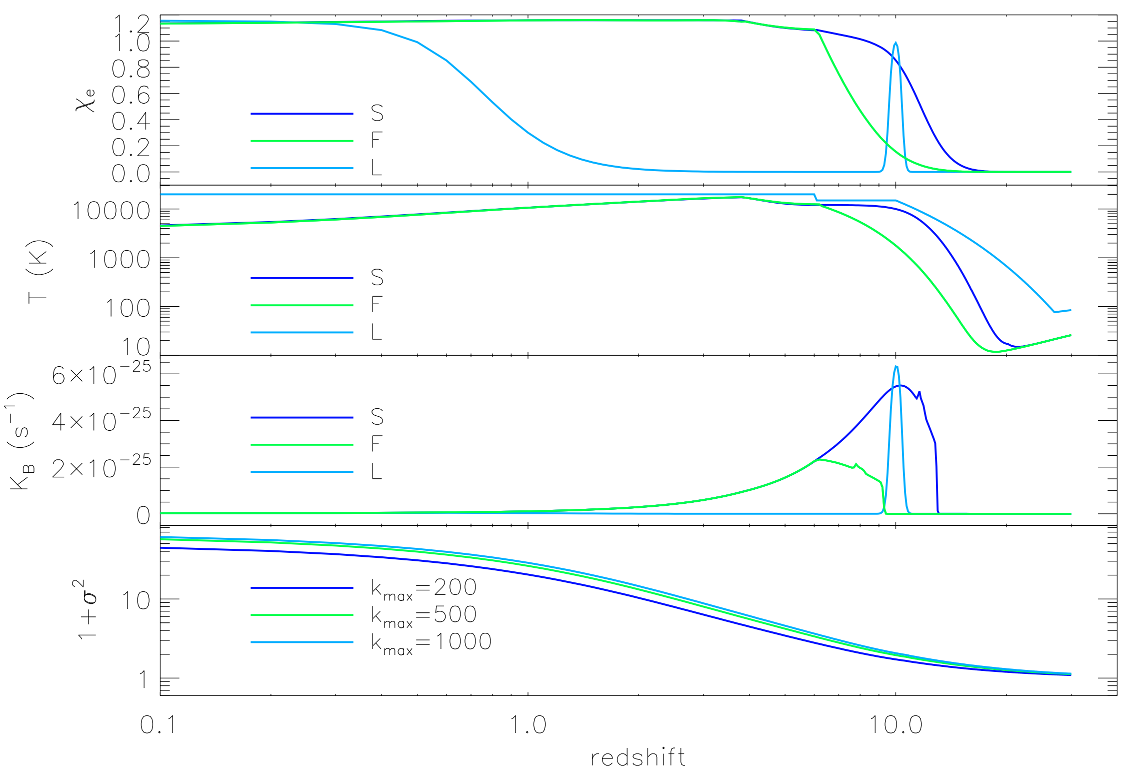

Figure 1: Redshift dependence of the electron ionization fraction (top panel), the electron temperature (top middle panel) and the bremsstrahlung rate (bottom middle panel) derived for the astrophysical and phenomenological reionization histories analyzed in this study. The bottom panel represent the clumping factor for three values of k_{max}, being in this approximation independent of the assumed reionization history.

The clumping factor has been evaluated following the analytical approximation we found in a previous work (see [12] for details about the cosmological approach and the analytical results) focused on different cosmological models, as well as on a wide set of the cut-off parameter k_{max}, in order to estimate the correction amplification factor to the free-free process, going from the one calculated under the cosmology assumed in the suppression and filtering models, and then generalizing it for every cosmological configuration.

Formally, we define a time dependent clumping variable as:

$$\sigma_{tot}^2 = 1 + \sigma^2 \; . \tag{18}$$

We then get the complete free-free term in a non homogeneous medium by multiplying the usual baryonic density parameter for this factor, i.e.:

$$\Omega_b^2 = \Omega_{b,homog}^2 \times \sigma_{tot}^2 \;, \tag{19}$$

where $\Omega_{b,homog}^2$ corresponds to the (standard) homogeneous case.

6 The code

The Fortran90 code, **FF_clump**, consists of different modules, namely:
- *constants* for the storage of the fundamental physical constants,
- *FrazIoniz* for the common variables management,
- *InpuFile* in which are defined the global functions for reading the parameters from the user input file,
- *functions* for the physical and cosmological functions that track the evolution of the primordial plasma,
- *clumping* which provide the approximated clumping factor based on the assumed cosmology and wavenumber limit.

The main program linking together all the modules is *FF_clump*, which contains the leading subroutine for the evaluation of the integral in Eq. (11), i.e. the free-free parameter y_B.

6.1 NAG routines

Basically, the free-free parameter y_B, in the redshift interval of interest, $z \in [0, 30]$, has been evaluated by means of the NAG routines, Fortran libraries developed for numerical or statistical analysis. Precisely, the integral in Eq. (11) is computed with the **D01AJF** routine, which calculates an approximation to the integral of a continuous function over a finite interval, based on the Gaunt 10-point and Kronrod 21-point rules. As will be explained in next section, the frequency range and the step number in redshift in which we divide the interval of interest is specified by the user.

6.2 Input file

The code is supplied with an input parameter file for the definition of the cosmological parameters, the output file names and other quantities describing the physics at the basis of the Universe's evolution. A typical example of the parameters set, selecting the case of the suppression reionization history, is provided below (note that also parameters relevant in other cases are included, but, of course, they have effect only when the corresponding case is selected).

Output_root is the prefix to every output file of each run:
output_root = supp
output_filename = FFandClump.dat

Primordial abundance of hydrogen:
hydrogen_fraction = 0.76

Cosmological parameters (default WMAP 9 yr):
$T_{cmb} = 2.725$
$h_{50} = 1.42$

$\Omega_b = 0.0463$
$\Omega_{cdm} = 0.233$
$\Omega_\Lambda = 0.721$
$\Omega_K = 0$
$\Omega_\nu = 0$

Reionization models: supp = suppression,
filt = filtering or late = late:
ReionHist = supp
$z_{re} = 13$

Late history parameters:
$m = 2 \cdot 10^{-4}$
$\beta = 9 \cdot 10^{10}$
$\varepsilon_0 = 1.3 \cdot 10^3$

Being $\phi = T_e/T_r$, ϕ_i is the initial electron temperature
for the evaluation of the free free distortion parameter:
suppression $\rightarrow 0.99999983$, **filtering** $\rightarrow 0.99999990$, **late** $\rightarrow 0.99999962$
$\phi_i = 0.99999983$

redshift at which occurred the heating epoch (z_h):
$z_h = 30$

Cosmological model associated to the correction factor
in the estimates of the variance (L0..L3, w0..w3) or NN (see below):
ClumpCosmoModel = L0

If adopting a different cosmological model set clump_cosmo_model = NN
and specify the corresponding value of σ_{corr}:
$\sigma_{corr} = 0$

Maximum wavelength at which perturbations are attenuated:
$k_{max} = 1000$

Frequencies grid extremes for which estimate the Gaunt factor.
Here $x_0 = \log(x(\lambda_{min}))$, $x_1 = \log(x(\lambda_{max}))$, $\lambda_{min} = 0.01$ cm, $\lambda_{max} = 1$ m:

$x_0 = 1.7$
$x_1 = -3.3$

Number of frequencies grid points:
GridNum = 501

Number of redshift grid points (typically $z_h * 1000$):
$z_{grid} = 30000$

Include/esclude clumping factor (boolean):
DoClumping = T

Relative accuracy of integration:
RelAcc $= 10^{-4}$

6.3 Output

The output of the code is split in two *.dat* files, *reion_saha* and *FFandClump*.

The file reion_saha stores the main quantities related to the specified reionization and electron temperature history, their corresponding chemical ionization fractions evaluated by the Saha equations, in a redshift grid from today to the heating epoch. Essentially, these variables describe the cosmic evolution of the quantities mostly relevant here. They are saved in the file sorted for increasing value of redshift and organized in the following order:

- redshift, χ_e, T_e, χ_H, χ_{H^+}, χ_{He}, χ_{He^+}, $\chi_{He^{++}}$, $K_{0B}(z)$, $K_B(z)$;

here the quantities χ_e and T_e are determined by the reionization history, the ionization fractions of the primordial H and He abundances are derived by the Saha equations, and the redshift dependent free-free rates are explicitly tabulated.

The file FFandClump contains the evaluated free-free parameter and its absolute error in the specified range of frequency values, thereby manages the variables describing the spectral distortions, such as:

- i, $\log(x)$, y_B, $\Delta(y_B)$,

being i a frequency grid counter, x the current dimensionless frequency value, $y_B(x)$ the free-free parameter on the whole integration time range considered, and $\Delta(y_B)$ its absolute error, as supplied by the NAG routine.

7 Results

The CMB spectrum, in terms of the brightness temperature, can be approximately described by the relation:

$$T_{br}(x) \simeq \left(\frac{y_B(x)}{x^2} - 2u\phi_i + \phi_i \right) T_r \ , \tag{20}$$

which depends on the frequency only (and holds at any redshift, provided that y_B and u are integrated over the corresponding interval $[z, z_h]$).

The resulting spectral distortions induced by the free-free including clumping are shown in Fig. 2, where each panel displays one of the possible scenarios accounted in the study, i.e. suppression (S), filtering (F) and late double peaked (L), and for three independent maximum wave-numbers k_{max}. We also report a curve, $k_{max}(nc)$, in which we do not account for the clumping amplification factor, aiming at directly unravel the impact of a primordial IGM density contrast on CMB spectral distortions. As expected, the net effect is stronger in the case of the astrophysical reionization models, where the cosmic plasma becomes fully ionized since redshift $z \simeq 10$, in agreement with an increasing clumping factor (see Fig. 1). Also, the two astrophysical models predict signal with relative differences much more smaller when clumping is included, since it introduces the most relevant amplification at $z \lesssim 6$ when the two scenarios predict almost complete ionization. Differently, the late history, in spite of being described by the same optical depth of the suppression model, is characterized by a first ionization peak taking place at $z \simeq 10$, where the clumping factor is almost negligible,

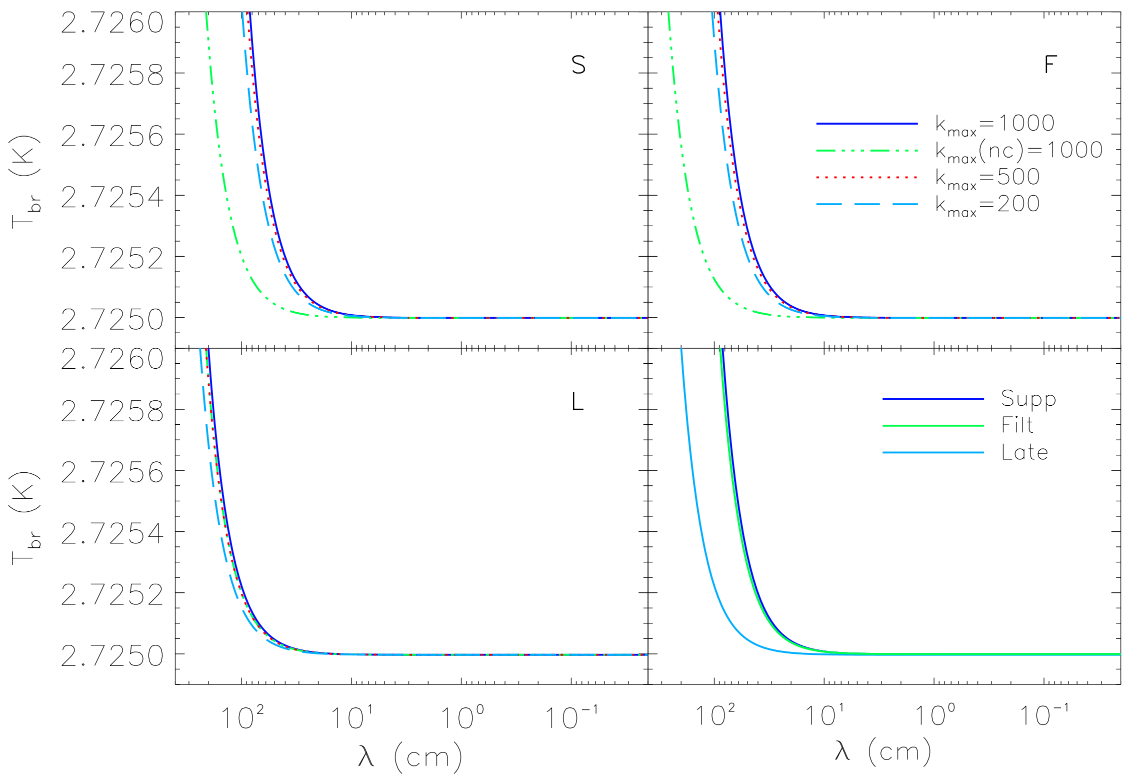

Figure 2: CMB spectral distortions induced by free-free for the three different reionization histories (see text for details) and for three values of k_{max}. The curve corresponding to $k_{max}(nc) = 1000$ (green dashed-triple dotted line) neglects the clumping factor, and is displayed in all panels for comparison with the case $k_{max=1000}$.

followed by a rapid ionization decrease resulting into an almost neutral medium till recent epochs ($z \simeq 1$), and then by a significant ionization increase only at low redshift when the clumping factor significantly increases. For this reason, in the phenomenological model the effective outcome of the clumping is much less outstanding, although we can again appreciate a tiny deviation of the CMB temperature between the case with (solid blue line) and without (dashed-triple-dotted green line) the inclusion of the clumping factor for an equivalent value of k_{max}. Finally, last panel in figure makes a comparison between the spectral distortions induced by these histories, reporting the (non preferential) case $k_{max} = 1000$. From this plot, it is easy to remark that the IGM density contrast noticeably affects the CMB brightness temperature evolution especially at decimeter wavelengths.

Basically, in order to describe the effect induced by free-free mechanism in terms of temperature excess at different frequencies, we can rewrite Eq. (20) as:

$$\frac{\Delta T_{ff}}{T_r}(x) \simeq \frac{y_B(x)}{x^2} \,, \tag{21}$$

where we have defined $\Delta T_{ff} = (T_{br} - T_r \phi_i)$.

In Fig. 3, we plot this dependence for the suppression scenario (upper panel) in the two limiting cases, corresponding to $k_{max} = 1000$ (with and without clumping) and $k_{max} = 200$, and for the three models (bottom panel) in the intermediate case $k_{max} = 500$. In the plots, the term $Avg_{k_{max}}$ in legend stands for a temperature variation averaged over the wavelengths from 1 cm to 1 m, which is the most relevant observational range for this process.

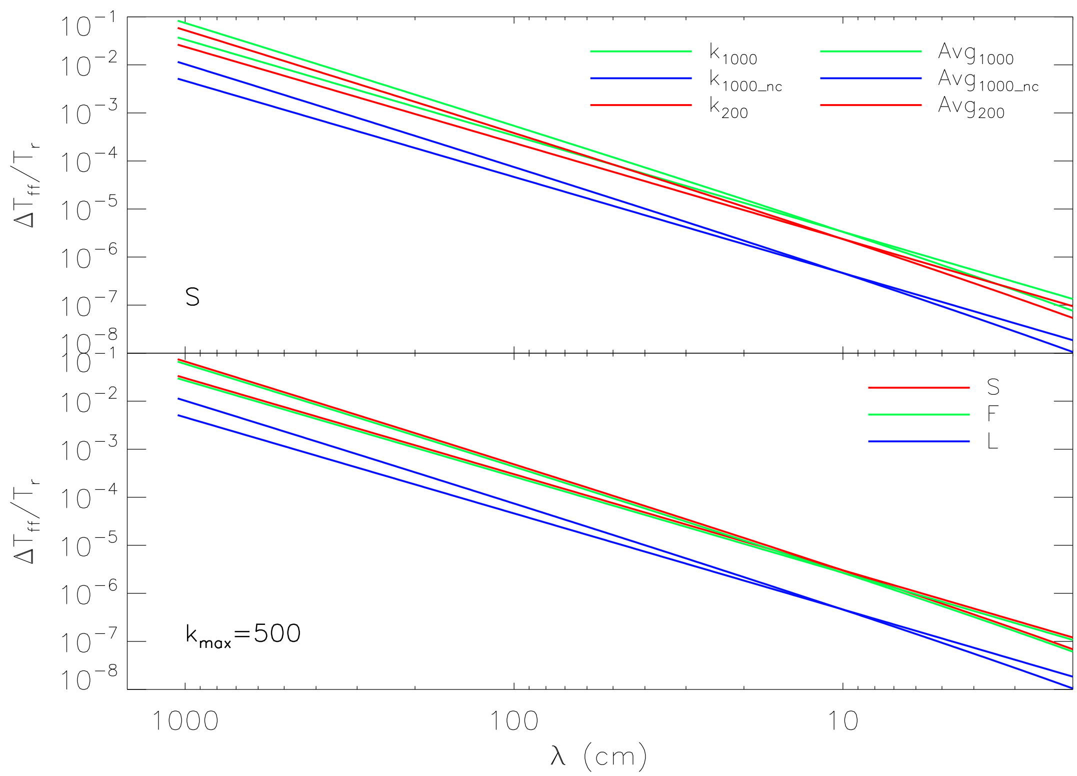

Figure 3: Relative variation of the brightness temperature as function of the wavelength assuming the $y_B(x)$ value properly evaluated at each frequency in the entire range, and a value $< y_B(x) >$ averaged in the interval (1,100) cm (denoted with Avg in legend). Upper panel: suppression model for $k_{max} = 1000$ (green line), $k_{max} = 200$ (red line) and $k_{max} = 1000$ without the correction due to the IGM density contrast (blue line). Bottom panel: comparison between the three reionization histories for $k_{max} = 500$. See also the text.

In all cases, the steeper lines refer to the complete computation, while the flatter lines are derived using the averaged $< y_B(x) >$ (for which the exact value depends also on the considered frequency range) and thus show the simple wavelength dependence $\propto \lambda^2$. The slope derived including all the effects is $\simeq 0.2$ steeper than that derived in the averaged case. Moreover, comparing the two panels appears, as expected, that the blue curve in top panel, corresponding to a null IGM density contrast with $k_{max} = 1000$, is highly comparable with the temperature variation generated by the late process for $k_{max} = 500$ as reported in lower panel, again denoting that the double peaked ionization history very weakly depends on the clumping factor, thus on the value of k_{max}, as stressed in previous section.

8 Conclusion

In this report, we presented a study of the CMB temperature spectral distortions induced in the primordial plasma by the bremsstrahlung process, in combination with a non negligible clumping factor due to a not null IGM density contrast. We developed the Fortran code **FF_clump** for the evaluation of the free-free distortion parameter assuming two astrophysical, suppression and filtering, and one phenomenological, late double peaked, reionization histories well known in literature. In the code, we implemented a routine aimed at finding a numerical solution of the Saha equations for a variable mixture of hydrogen and helium primordial abundances, providing a precise evaluation of their corresponding ionization states. We also performed a dedicated routine for the relative Gaunt factors. In particular, the code is provided with an input file where the user can define all the parameters set necessary for each run, as we explained extensively in the report, elucidated with a typical example.

We showed, in a redshift interval $z \in (0, 30)$, the electron ionization fraction and the electron temperature for the histories accounted in the code, the corresponding bremsstrahlung rate and the clumping, the correction factor to the free-free term which accounts for the IGM density contrast. We compare the effect on free-free distortion coming from different values of k_{max} for the same reionization history and for different scenarios. Focussing on the wavelength dependence on the free-free excess, we compare the signal slope derived from the complete computation and assuming an $y_B(x)$ value averaged over a suitable wavelength range (from 1 cm to 1 m), obtaining a somewhat larger slope in the former case than in the latter.

The late double peaked model turns out to be characterized by a weak impact of clumping factor related to the peculiar shape of the ionization fraction evolution, from a fully ionized Universe at $z = 10$ to a nearly neutral one between $1 < z < 10$ where the clumping factor becomes remarkable. The two astrophysical histories show, on the other hand, much less relative different free-free distortions when the clumping factor is included, because an almost full ionization state is achieved in both cases when the clumping factor significantly amplifies the signal.

Acknowledgements − We acknowledge support by MIUR through PRIN 2009 grant n. 2009XZ54H2 and by ASI through ASI/INAF Agreement I/072/09/0 for the *Planck* LFI Activity of Phase E2.

References

[1] Burigana, C., Danese, L., de Zotti, G., 1991, Astronomy and Astrophysics, vol. 246, p. 49

[2] Burigana, C., de Zotti, G., Danese, L., 1991, Astrophysical Journal, vol. 379, p. 1

[3] Burigana, C., de Zotti, G., Danese, L., 1995, Astronomy and Astrophysics, vol. 303, p. 323

[4] Danese, L., De Zotti, G., 1997, La Rivista del Nuovo Cimento, vol. 7(3), p. 277

[5] Kompaneets, A.S., 1956, Zh.E.T.F. vol. 31, p. 876 [Sov. Phys. JETP, vol. 4, p. 730 (1957)]

[6] Procopio, P., Burigana, C., 2009, Astronomy and Astrophysics, vol. 507, p. 1243

[7] Rybicki, G.B., Lightman, A.P., 1979, *Radiative processes in astrophysics*, Wiley, New York

[8] Salvaterra, R., Burigana, C., Schneider, R., Choudhury, T.R., Ferrara, A., Popa, L.A., 2009, Mem. S.A.It., vol. 80, p. 26

[9] Trombetti, T., Burigana, C., 2011, Internal Report IASF-BO 589/2011

[10] Trombetti, T., Burigana, C., 2011, Internal Report IASF-BO 590/2011

[11] Trombetti, T., Burigana, C., 2011, Internal Report IASF-BO 591/2011

[12] Trombetti, T., Burigana, C., 2011, Internal Report IASF-BO 594/2011

[13] Trombetti, T., Burigana, C., 2012, Journal of Modern Physics, vol. 3, p. 1918